

The Journal of **Ancient Egyptian Architecture**

vol. 2, 2017

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Cite this article:

G. Dash, 'Occam's Egyptian razor: the equinox and the alignment of the pyramids', *JAEA* 2, 2017, pp. 1-8.

JAEA

ISSN 2472-999X

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Occam's Egyptian razor: the equinox and the alignment of the pyramids

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The builders of the Great Pyramid of Khufu aligned the great monument to the cardinal points with an accuracy of better than four minutes of arc, or one-fifteenth of one degree.¹ The Great Pyramid's neighbor, the Pyramid of Khafre, is aligned with an error of approximately 6 minutes, one tenth of one degree.² The builders of Snefru's Red Pyramid at Dahshur achieved an accuracy of 8.7 minutes.³ All three pyramids exhibit the same manner of error; they are rotated slightly counterclockwise from the cardinal points.

How the Egyptians managed to achieve such accuracy has long been debated. In recent years, four of the candidate methods have been tested and found workable.⁴ These include the pole star method proposed by Flinders Petrie,⁵ the circumpolar star method tested by Joseph Dorner,⁶ the simultaneous transit method proposed by Kate Spence,⁷ and the solar gnomon shadow method suggested by Martin Isler.⁸ Yet there is one straightforward method that scholars have largely ignored, perhaps because it was thought to lack any hope of achieving the requisite accuracy.⁹ This is the 'equinoctial solar gnomon method'. It uses a vertical rod to track the movement of the sun on the equinox.¹⁰

The solar gnomon or 'Indian circle' method

The equinoctial solar gnomon method is a variant of the solar gnomon method suggested by Martin Isler.¹¹ The solar gnomon method is sometimes referred to as the Indian circle method, because it was thought to have been used on the Indian subcontinent.¹² In the solar gnomon method, a surveyor starts by placing a rod into the ground as shown in figure 1. The rod is known as a gnomon. As the sun rises in the east, the gnomon projects a shadow to the west. The surveyor watches

1 Dash (2015b), p. 11.

2 Dorner (1981), p. 80.

3 Dash (2015a), p. 359.

4 Dash (2015a), pp. 351-364.

5 Petrie (1883), pp. 211-212.

6 Dorner (1981), p. 146.

7 Spence (2000), pp. 320-324.

8 Isler (1989), pp. 197-199.

9 Magli (2013), p. 90.

10 A well-known method proposed by Edwards (Edwards (1947), pp. 250-251) uses a circular wall around an observer to identify true north. It was not included in this analysis because it has not been field tested.

11 Isler (1989), pp. 197-199.

12 Isler (1989), p. 197.

the shadow and at regular intervals marks the position of the tip of the shadow on the ground. As the day progresses, the surveyor's markings should, in theory, form a smooth curve. The curve will bend around the gnomon in the summertime and away from it in the wintertime. The curve is known as the declination line or more commonly, the shadow line.

At the end of the day, the shadow line being complete, the surveyor takes a string, places it over the gnomon and rotates the taut string around the gnomon, describing a circle which intercepts the shadow line at two points. These points lie on an east-west line.

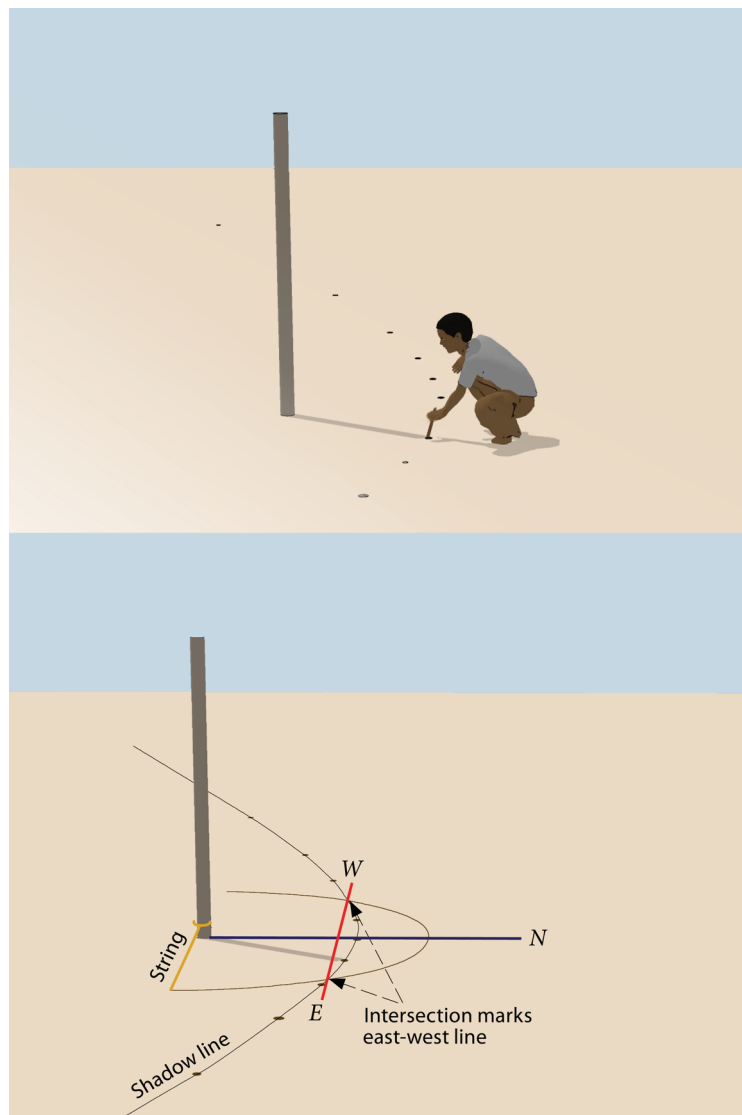


Fig. 1. The solar gnomon or Indian circle method. The shadow line shown is typical of that formed in the summertime. In the wintertime, the shadow line curves away from the vertical rod, or gnomon.
(Illustration by Wilma Wetterstrom)

The equinoctial solar gnomon method

The equinoctial solar gnomon method is simply the Indian circle method used on the equinox (fig. 2). On the equinox, the surveyor will find that the tip of the shadow runs in a straight line and

nearly perfectly east-west. Since the shadow line is already straight and already runs east-west, the second step in the solar gnomon method, drawing a circle around the gnomon, is not needed.¹³

Testing the equinoctial solar gnomon method

To test the equinoctial solar gnomon method, I built a 0.91 meter by 6.10 meter (3 foot by 20 foot) wooden platform at my home in Pomfret, Connecticut and roughly leveled it (fig. 3). I set the gnomon along the platform's midline near its southern edge. The gnomon was built from a 3.2 cm (1.25 inch) diameter wooden dowel rod capped with wooden half ball. A metal pin was inserted in its top.¹⁴ Vertical 5 cm by 10 cm (2 by 4 inch) risers were used to suspend the rounded, wooden top of the gnomon 83 cm over the surface of the platform.¹⁵

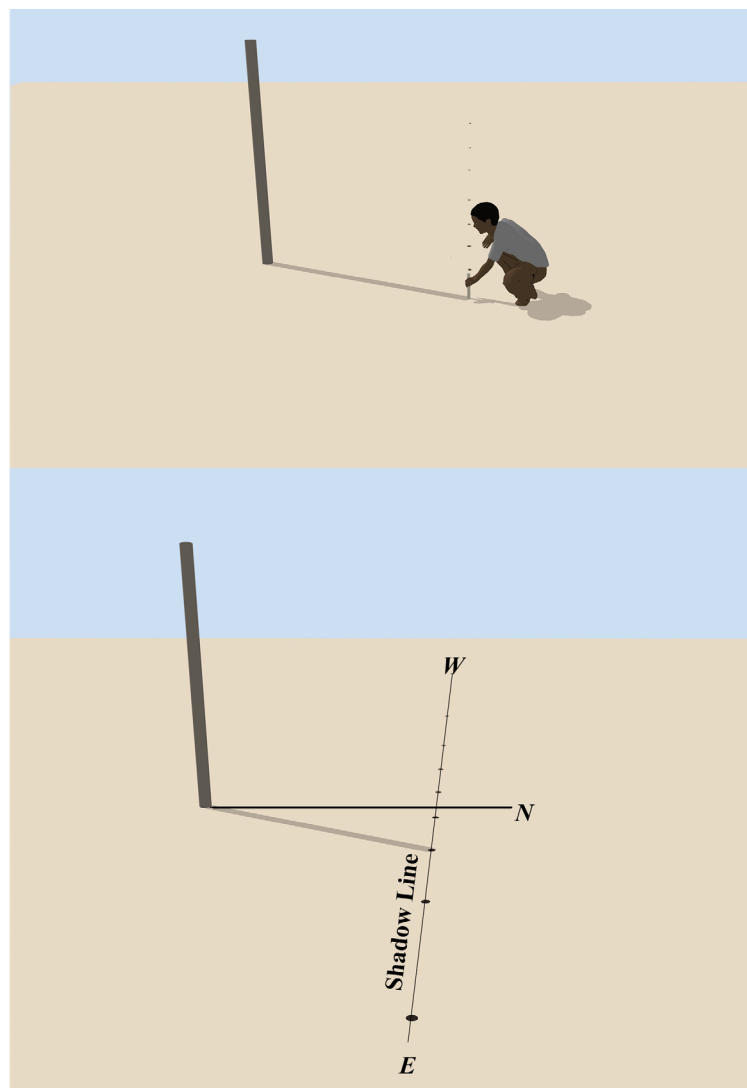


Fig. 2. On the equinox, the shadow line runs in a straight line, very nearly east-west. (Illustration by Wilma Wetterstrom)

¹³ The Egyptians could have established the day of the equinox by observing the solstice and counting forward 91 days.

¹⁴ The metal pin is not needed as part of the equinoctial solar gnomon test. It is used in the Indian circle method.

¹⁵ The design of the gnomon was a product of experiment. Dowel rods of differing diameters and different types of caps, including conical ones, were tried. The dowel rod and half round ball combination produced the most workable shadow.

I began the test at 8:04 am on September 22, 2016, the day of the autumnal equinox. Curiously, the gnomon's shadow exhibited a central core that was slightly brighter than the rest of the shadow (fig.4). I used the far edge of this central core to track the shadow's movement. I marked its place every minute or so.¹⁶



Fig. 3. The equinoctial solar gnomon method was tested on this platform, here viewed from the northeast. In the center of the platform is the gnomon, set along its midline on its southern edge.

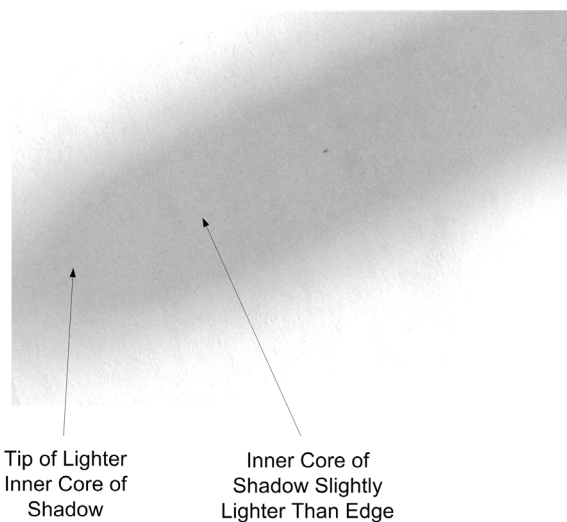


Fig. 4. The shadow produced by the gnomon is, curiously, slightly brighter in its central core. The tip of the central core was used to track the shadow's movement.

¹⁶ Dash (2016b) (<http://www.DashFoundation.com/OCCAMMOV.MOD>). This video shows the marking of the tip of the shadow as it moves.

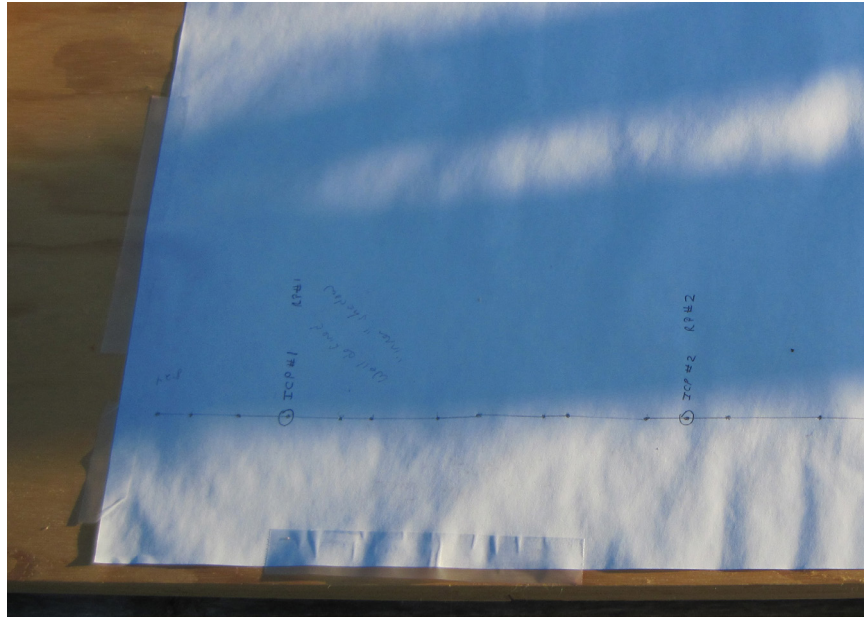


Fig. 5. Shown in this photo, taken in the late afternoon, is a portion of the morning's data. The two points circled were used to evaluate the method.

By 8:24 am, I had tracked the movement of the gnomon's shadow along a 63 cm long path. As expected, the line ran relatively straight. I circled two points along the shadow line where the points were particularly well aligned (fig. 5). Later I would use these two points to test the accuracy of the method.

I followed the same procedure in the afternoon. At the end of the day, I had four points circled, two on the east side of the platform and two on the west, which would be used in my evaluation.

Previous to the tests, I had established a local control grid using a total station. I aligned the grid with due north by first focusing the total station's telescope on Polaris and then loading Polaris' exact location at that time into the total station. I verified the alignment's accuracy by checking the location of Kochab. The total station's read out of Kochab's position was accurate to within ten seconds of arc. I then set a control point to the east of the wooden platform and randomly assigned it a location of N=1000 meters and E=5000 meters. This would serve as the origin of my control grid.

I used the total station to establish the exact locations on my control grid of the four points I had circled (table 1).

Point Number	Northing (m)	Easting (m)	Elevation (m asl = meters above mean sea level)
1	1004.477	4938.173	178.120
2	1004.475	4938.001	178.120
3	1004.487	4942.992	178.124
4	1004.486	4943.081	178.124

Table 1. Measured data.

I then calculated the angle of the lines formed by pairs (1, 3) and (2, 4) with respect to due east-west (table 2):

Point Pair	Northing Distance Between Points (m)	Easting Distance Between Points (m)	Azimuth angle with Respect to due East-West (minutes) ¹⁷
1, 3	.010	4.819	-7.13
2, 4	.011	5.080	-7.44

Table 2. Evaluation of derived East West lines.

The mean error of the two lines was -7.3 minutes of arc. This represents a slight counterclockwise rotation from cardinal points. This type of error was expected. The earth's tilt with respect to its orbital plane around the sun, the 'declination', changes over the course of a day. In theory, this change produces an error of -5.9 minutes of arc on the autumnal equinox.¹⁸ Thus the test produced results within 1.5 minutes of the expected value.¹⁹

The mean error of the three pyramids mentioned above, Khufu, Khafre and the Red Pyramid of Snefru, is -6.2 minutes. The magnitude and direction of these errors suggest that it is possible that all three pyramids were aligned using the equinoctial method on the autumnal equinox.

Comparing Fourth Dynasty pyramids

Table 3 shows the alignment of six pyramids of the Fourth Dynasty with respect to cardinal points. Note that the largest pyramids are the best aligned, and their corners are the best squared. Clearly, the Egyptians took greater care when building their largest pyramids, both in squaring the corners and aligning the monuments with cardinal points. In particular, the problems the Egyptians encountered in building the pyramid at Meidum and the Bent Pyramid at Dahshur may have taught them that large pyramids needed well-formed and well-aligned foundations. Later pyramids such as the Pyramid of Menkaure tended to be smaller, and these pyramids may not have required such well-ordered foundations. For these, the Egyptians may have simply economized.²⁰

Extending the line

The equinoctial solar method described here produces two points on the ground about 5 meters apart. To lay in a baseline for a pyramid, the Egyptians would have had to extend the line formed by these two points for hundreds meters with little error. Several methods have been proposed by which the Egyptians might have achieved that.²¹

The Egyptians could also have started with a larger gnomon, but they would also have needed a larger platform on which to trace the shadow, and one which was precisely leveled. Such a platform

¹⁷ Surveyors measure azimuths clockwise from true north. A clockwise rotation with respect to cardinal points is reported as positive, a counterclockwise rotation, negative. In this case the difference is given with respect to the true East-West line.

¹⁸ On the vernal equinox, the expected error is of the same magnitude but opposite in direction: +5.9 minutes of arc.

¹⁹ This small error could be explained by the imperfect leveling of the platform which was 4 mm higher on the east than west.

²⁰ Lehner also speculates that a shift in emphasis away from the pyramid and toward the temples may explain the diminutive size of the Menkaure Pyramid (Lehner (1997), pp. 134-135). Petrie notes that the base of the Pyramid of Menkaure is highly irregular (Petrie (1883), pp. 110-111).

²¹ Dash (2014).

would have been available, however, as the platform around the Great Pyramid is leveled to within a few centimeters over its entire 920-meter periphery.²²

Pyramid	Date (B.C.)	Source	N	E	S	W	Average
Meidum	2522	Petrie ²³	-35.4	-20.6	-23.6	-18.1	-24.4
Bent	2507	Dorner ²⁴	-7.5	-17.3	-4.2	-11.8	-10.2
Red	2496	Dorner ²⁵		-8.7			
Khufu	2476	Lehner/ Goodman ²⁶	-2.9	-3.4	-3.7	-4.6	-3.6
Khafre	2444	Dorner ²⁷	-5.2	-6	-5.7	-6	-5.7
Menkaure	2411	Petrie ²⁸	16.8	12.4	13.0		14.1

Table 3. Alignments of the casing sides of selected pyramids with respect to cardinal directions (angles in minutes).

Conclusion

The equinoctial solar gnomon method appears to be workable. It joins the list of methods the Egyptians might have used to align their pyramids.

As to the methods they actually did use, the Egyptians, unfortunately, left us few clues. No 'engineering documents or architectural plans have been found that give technical explanations demonstrating how the ancient Egyptians aligned any of their temples or pyramids. No Egyptian compasses have ever been discovered, nor has any other type of sophisticated survey equipment'.²⁹ The records that do survive consist primarily of descriptions of foundation ceremonies for important buildings.³⁰ However, it is unclear as to what extent these descriptions describe technical details as opposed to the ceremonies themselves.

Nonetheless, among our choices, the equinoctial solar gnomon method has a certain appeal. It produces results that match the actual alignments of the largest pyramids of the pyramid age in magnitude and direction. It is also the 'Occam's Razor' candidate. It is hard to imagine a method that could be simpler either conceptually or in practice.³¹

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²² Cole (1925), pp. 3-5.

²³ Petrie (1892), p. 6.

²⁴ Dorner (1986), p. 51.

²⁵ Dorner (1998), p. 23.

²⁶ Dash (2012), p. 19.

²⁷ Dorner (1981), p. 80.

²⁸ Petrie (1883), p. 111.

²⁹ Nell and Ruggles (2014), p. 305.

³⁰ Belmonte, Polo and Miranda (2009), pp. 197-206.

³¹ Correction to this paper, if any, can be found at Dash (2016a).

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