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An elegant vault design principle identified in Old and New Kingdom architecture

David Ian Lightbody & Franck Monnier

Excavations[1] in Egypt have occasionally brought to light markings and inscriptions relating to the 'intrados' (the lower or inner curve of an arch or vault). Although poorly preserved, these few surviving texts have preserved valuable information regarding the preparation and construction of certain types of vaults (fig. 1).

Researchers have tended to approach the analysis of these fragments by linking the forms traced on them to precise geometric constructions, without first establishing the legitimacy of such an approach. Elliptical arches are often referenced, without first demonstrating that the ancient Egyptians were able to generate such complex forms. Even if they were not aware of abstract concepts such as ellipses and catenary arcs, the ancient Egyptians may nevertheless have been able to apply them using rudimentary processes intended to produce shapes with the approximate form of an oval, or with the inherent strength of a catenary arch. There are a number of different plausible scenarios, but certain hypotheses, without ever having been proven, seem to have acquired the status of established fact.

Studies of the design of mudbrick barrel vaults and inclined-layer Nubian arches have lead researchers to investigate how the forms of vaults were first calculated and how their profiles were traced out, either using geometric rules or empirical, practical processes. In one of the earliest publications dealing exclusively with questions of construction in ancient Egypt, Auguste Choisy proposed a simple design method which could have been used for making Nubian vaults.[2] The profiles of Nubian vaults resemble the raised handles of baskets, and he proposed a method of defining a circular arc which can be used to create this type of vault. He called this the Egyptian basket-handle type,[3] a name subsequently adopted by others.[4]

El-Naggar subsequently noted that while vault forms can often be shown to be close to certain theoretical vault types, as Choisy had done in this case, they can only rarely be shown to be precisely comparable.[5] This disconnect between theoretical models and surviving vaults does not, of course, mean that no precise rules were used for their construction, and herein lies the difficulty of determining what geometric rules and methods were employed by the ancient Egyptians.

1 The authors would like to thank Andrew Conner for sharing his research into the Saqqara ostracon. An archived version of his website, which was devoted to the Saqqara ostracon and his analysis, is currently available at the time of writing at: http://web.archive.org/web/20070708153336/http://goldenlot.us/ADC/ostrakon.htm [date accessed: 15 June 2017]. We gratefully acknowledge Alain Guilleux for providing the photos for the article.

2 Choisy (1904), pp. 46-48.

3 Choisy (1904), p. 46.

4 El-Naggar (1999), p. 363.

5 El-Naggar (1999), p. 363.

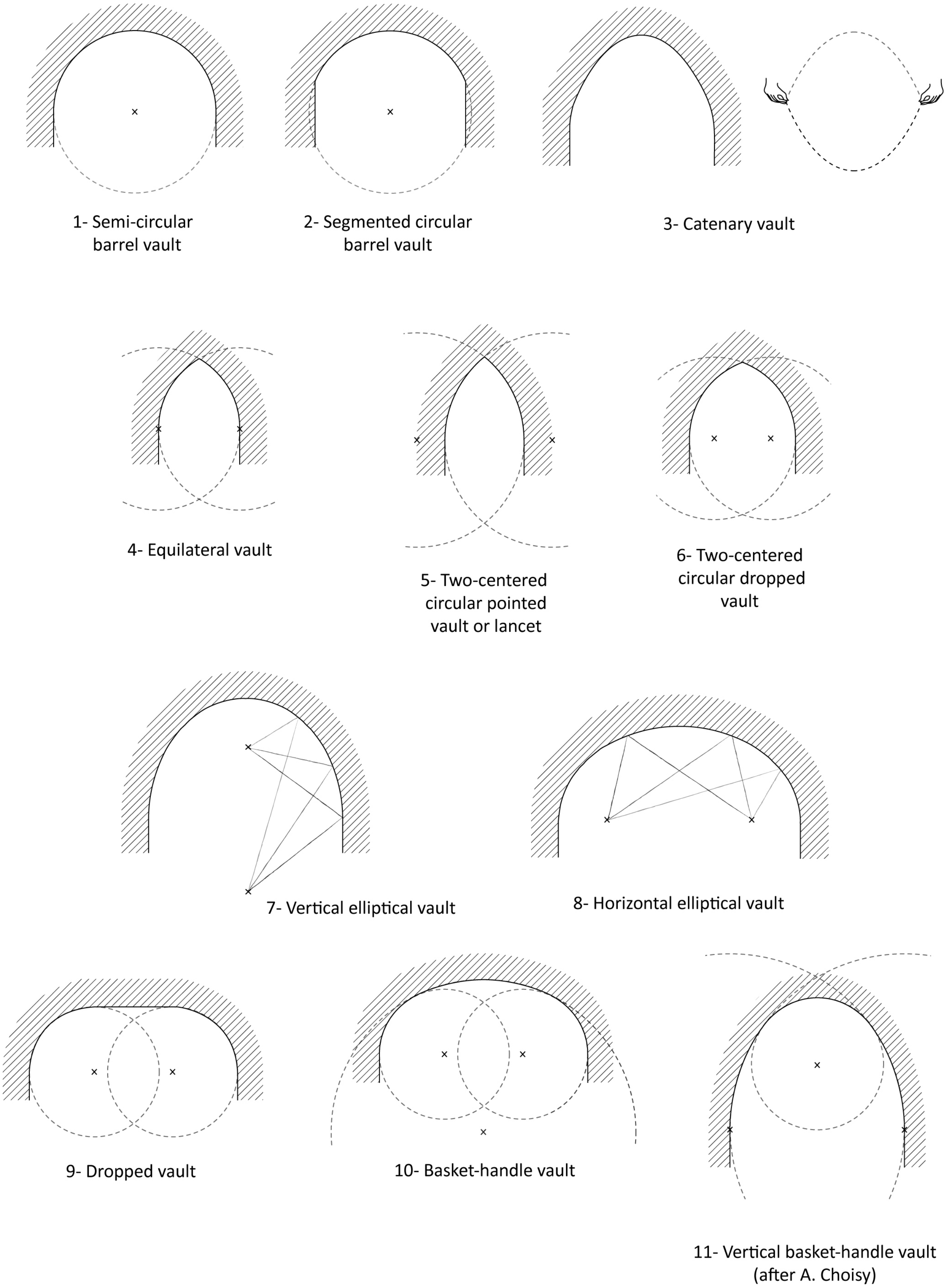

Fig. 1. Diagrams depicting types of vaults relevant to the current analysis.

Excavation reports, for the most part, record vault forms with appropriate precision, but not of a level that would allow researchers to draw clear geometric lessons from the evidence. In some cases, the degree of similarity between barrel vaults, catenary vaults, and elliptical vaults means that only a few centimeters of difference separate the theoretical forms, so that the small details can be significant.

Moreover, less well-defined vault profiles suggest that the Egyptians often worked by habit. The construction of mud-brick vaults became so familiar that they eventually produced them free hand, further complicating the analytical process.[6]

In the case of stone built structures, however, more care and accuracy would be required, given the effort involved in the initial construction and the difficulty of altering the form after it had been made. In the case of rock-cut chambers the curved form of a vault could not simply be traced out in advance as no end walls would be available on which to draw the form, before the vault had been cut. The curve would therefore have been drawn out nearby and the vertical heights at each horizontal position across the vault marked out in order to guide rock cutting into the ceiling to appropriate depths.

With these caveats in mind, in this article we present and compare two analyses of important ancient Egyptian geometric profiles. The first analysis is of a sketched profile found beside KV9. This was first published by Franck Monnier in French.[7] The second analysis is of an arch sketched on a limestone ostracon from Saqqara, which was first completed by Andrew Conner, a naval architect and chartered engineer. He previously published the analysis on a website dedicated to the ostracon.[8]

The reason for bringing these two analyses together is that the same, distinctive, geometric method, based on a 3-4-5 triangle, may have been used to create both profiles. As well as describing the geometry of the method, we also demonstrate that the method fits both architectural contexts very well. This reinforces the strength of the conclusions that can be drawn from the evidence.

Tracing of a vault in proximity to the tomb of Ramses VI (KV9, 20th Dynasty)

Following the clearance of the tomb of Ramses VI in the Valley of the Kings, Georges Daressy uncovered a large sketched profile of a vault drawn out on a wall close to the entrance of the tomb in black ink (fig. 2-[1]).[9] He found that it was executed after dressing and lime-whitening of the wall surface. At the time it was discovered the diagram had already suffered from erosion and was incomplete. Today it has almost completely disappeared.[10] After studying the sketch, Daressy noted that the width of the horizontal line at the base of the diagram (6.334 m) was almost identical to the width of the vault which formed the hypogeum in the adjacent tomb of Ramses VI (6.35 m).[11] It was a logical step to propose that the diagram was intended to guide the construction of the excavated vault in that tomb. However, if that was intention, the final height of the chamber deviates from the described form, despite the fact that the walls were finished and decorated.[12] The final

6 The various forms of Nubian vaults at the Ramesseum are discussed in Goyon *et al.* (2004), p. 128, fig. 132-d and Monnier (2015b).

7 Monnier (2015a).

8 http://web.archive.org/web/20070708153336/http://goldenlot.us/ADC/ostrakon.htm [date accessed: 15 June 2017].

9 Daressy (1907), pp. 237-241.

10 El-Naggar, (1999), p. 333, no. 1494.

11 Daressy (1907), p. 238.

12 Theban Mapping Project tomb KV9 http://www.thebanmappingproject.com/sites/pdfs/kv09.pdf.

form of the tomb's vault is significantly flattened and is more reminiscent of a dropped vault, a type of chamber roofing form that is seen elsewhere, including in the majority of the tombs in the Valley of the Kings. Only the funerary chamber of the more ancient tomb of Ramses III (KV11), situated not far from the sketch's location, at around 50 m distant, contains a vault that very closely approaches[13] the form of the curve described by the profile and the dimensions of the diagram (fig. 2-[3]). The horizontal line on the diagram is marked approximately every 0.146 m.

Daressy assumed a standard cubit *mḥ nswt* of 0.5277 m was used in the design, and he then devised a method (fig. 2-[2]) whereby an elliptical profile resembling the sketch could have been obtained.[14] Here is the (rather complex) procedure:

1. Trace out a horizontal center line AB of 13 1/3 cubits.
2. From the mid-point of AB, at O, trace out a perpendicular vertical line with a height of 5 1/3 cubits.
3. From point C at the top of the vertical, make a semi-circle with a radius of 6 2/3 cubits, cutting AB at points E and D.[15]
4. Make an ellipse using E and D as the two foci and C as the vertex. Employ the 'gardener's method' to create the ellipse. The curve should start at A and end at B.
5. Draw out the horizontal 'spring line' for the vault 3 cubits below point C. The width of this line will be 12 cubits and will equal the span of the vault.

With his procedure Daressy also seems to have employed triangles having the same proportions as a 3-4-5 triangle; the triangles COD and COE, each having dimensions 4, 5 1/3, and 6 2/3 cubits.[16] These measures correspond to values 3-4-5 increased by a factor of 1 and 1/3, thus maintaining the notable proportions.

On this basis, Daressy hypothesized that the workers applied the so-called gardener's method, which consists of planting two sticks on flat and level ground at foci E and D, attaching a rope (in this case of 13 1/3 cubits) between them, and sliding a third stake along inside it while keeping the rope sufficiently taut on either side, in order to obtain the desired ellipse.

Daressy's hypothesis and the conclusions he drew are often accepted without reservation, to the extent that certain elements of his analysis are now considered to be established fact.[17] The reality is that Daressy demonstrated nothing concrete, and proposed a method of construction which was not founded on proofs. The form of the upper section of Daressy's ellipse very closely approximates the form of the drawing traced out near the entrance to the tomb of Ramses VI, however, and as Daressy also noted, the ellipse and the drawing sketched out on the wall diverge at several points. In conclusion, there is no clear evidence that Daressy's method was known to the ancient Egyptians.

The current authors attempted to establish if the curve could have been created using a more straightforward method based on a circular arc. Elementary rules of geometry were first employed to determine the radius of the circle passing through three points: the two end points of the horizontal line of 12 cubits, and the top of the arc at C. It turned out that the radius of a circle passing

13 Theban Mapping Project tomb KV11 http://www.thebanmappingproject.com/sites/pdfs/kv11.pdf.

14 We doubt the basis of such levels of precision (to a tenth of a millimeter) attributed to the cubit, however, the value does fall within a range that is historically appropriate for the period. See Carlotti (1995), p. 138.

15 In step 3 described above, the value of 6 1/3 cubits provided in the article by Daressy has been corrected to 6 2/3 cubits, which corresponds to the 3.5182 m stated. The analysis in the remainder of his article seems to support this correction.

16 The final pair of values were erroneously recorded again by Daressy as 6 1/3 cubits. See Daressy (1907), p. 238.

17 Arnold (1991), p. 22; El-Naggar (1999), p. 333; Rossi (2003), pp. 114-115.

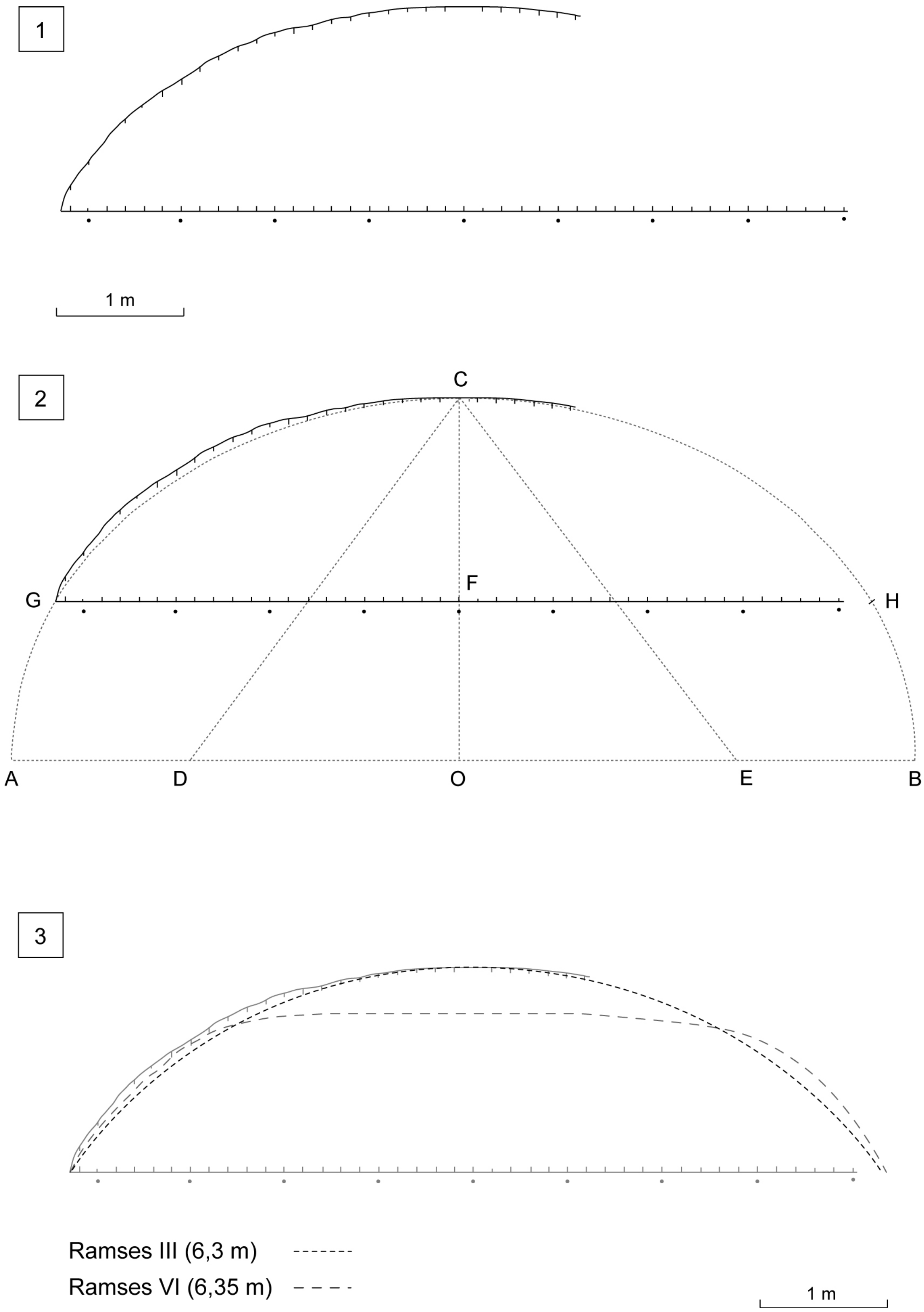

Fig. 2.
1. Diagram of a curve represented on the wall of the excavated trench near tomb KV9 of Ramses VI (Daressy 1907, p. 241, fig. 1).
2. Daressy's method to draw the curve of tomb KV9.
3. Comparison of the drawing at the tomb of Ramses VI with the profile of vaults surmounting the funerary chambers of Ramses VI (tomb KV9) and Ramses III (tomb KV11). Widths of the vaults are given.

through these points is very precisely equal to 7 ½ cubits. This is a remarkable value, and all the more remarkable because it implies that the triangle on which the arc is based has sides measuring 4 ½, 6, and 7 ½ cubits, and a hypotenuse which is the radius of the circle. This triangle, again, possesses exactly the same proportions as a 3-4-5 triangle.

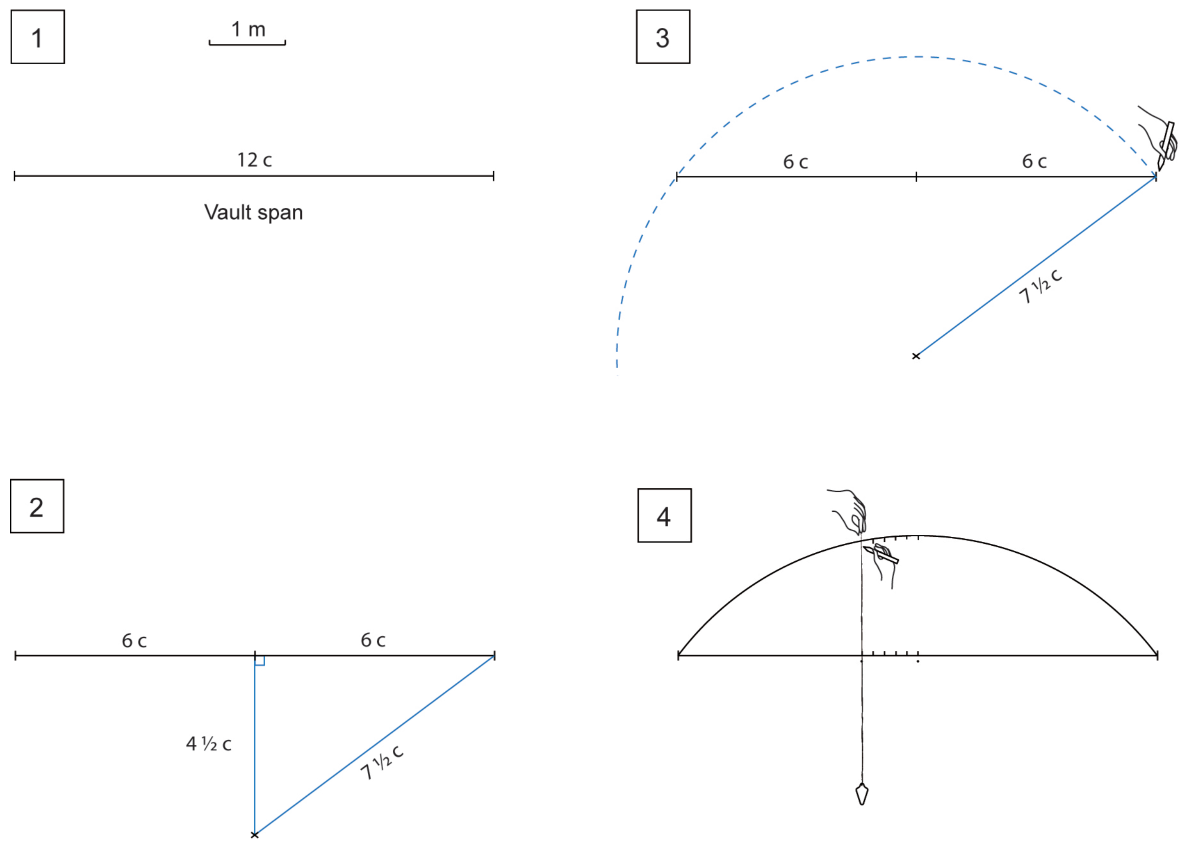

Fig. 3. Method devised by F. Monnier to construct the profile of a vault which matches that discovered during the excavation of the tomb of Ramses VI, and also the vault over the burial chamber in the tomb of Ramses III.

The circular arc obtained in this way also corresponds precisely with the form of the vaulted roof which covers the funerary chamber of Ramses III (fig. 2-[3]).

With this circle and this method of curve construction we have discovered an alternative, and much simpler, procedure to that proposed by Daressy. From a technical point of view our method has the advantage of being based on measurements which are easily obtained within the ancient Egyptian system of linear measurement. Its most interesting geometric characteristic is that it employs a triangle having the same proportions as a 3-4-5 triangle, for which the hypotenuse is also the radius of the circle.

The workers who carried out the construction of vaults on-site built structures according to instructions provided by their master of works. In turn, the master of works would have followed a vault design process that was determined by the internal characteristics of the architectural space to be covered, in this case the burial chamber of the tomb. The vault was therefore dimensioned according to the spatial characteristics of the tomb. The first parameter required to define the vault profile was, therefore, the width of the space to be covered, in other words the span of the vault.

In this case the width is equal to a whole 12 cubits. The first parameter used in Daressy's method is, contrastingly, the major axis of the ellipse, which is 13 1/3 cubits. With the mathematical tools at their disposal, it is difficult to imagine how the ancient Egyptians could subsequently have established the length of rope or cord required to produce an ellipse that would accurately run through points G, C, and H, and for which the distance between G and H was 12 cubits. This is an extremely complex problem, even more so when attempted within the context of ancient Egyptian mathematics.

The method of construction based on a circular arc is much simpler in this respect. First, the half width of the vault would be easily calculated, in this case 6 cubits. The engineer would then determine the sides of a triangle with proportions 3-4-5, where the side 4 would equate to the 6 cubits. The multiplication factor is therefore 1 ½, and so they would subsequently find that the other side lengths to have values of 4 ½ and 7 ½ cubits respectively (fig. 3). At that stage it only remained to trace out a circle with radius of 7 ½ cubits, running from the end points of the so called 'spring line'.

With respect to other possible techniques employed, the catenary method provides a fairly simple practical way to trace out the requisite profile, avoiding all mathematics in the process. If we suspend a cord or chain between two points at the same vertical height, with the ends separated by 12 cubits, the shape of the hanging mass will, due to its own regularly distributed weight, describe a catenary curve.

For vaults of this approximate form, the profiles of a catenary curve, a vault with a circular arc, and an elliptical vault can be remarkably similar. No document describes the actual process or processes used, so the diagrams could indeed represent catenary curves produced using a manual process. It is therefore not necessary to resort to using a complex mathematical method involving an ellipse, constructed using the gardener's method, to explain the profile. Neither the gardener's method nor a form which is indisputably an ellipse is attested in the ancient Egyptian culture.

Daressy claimed that the portion of the ellipse he described is so similar to the curve traced at the entrance to the tomb that we can assume that any differences are due to construction errors.[18] In this article, however, the current authors have proposed that an arc of a circle also closely matches the profile of the curve traced out at the entrance to the tomb, and that there is therefore no reason to prefer a hypothesis based on an ellipse. It is more complicated and is not attested elsewhere in the ancient Egyptian cultural material. A hypothesis based on a catenary curve should not be ruled out, even if its form deviates rather more from the curve drawn at the tomb entrance than do the forms produced by the other methods.

In conclusion, the characteristics of the circular arc which has been described in the first half of this article seem so noteworthy that it would be remarkable if they had occurred by accident. The current authors consider that the circular arc is very likely the geometric form used in the construction of the sketched curve at KV9.

The Third Dynasty traced vault profile on ostracon JE 50036

The second major piece of evidence referenced in this article is a limestone ostracon with a delineated profile of a vault illustrated on it that was discovered in the complex of pharaoh Djoser, beside the north pavilion. It is currently on display in the Imhotep museum at Saqqara (JE 50036).[19]

18 Daressy (1907), p. 238.
19 Firth (1924), p. 122.

Fig. 4. Ostracon dating to the 3rd dynasty
(JE 50036, previously in the Cairo Museum, now housed in the Imhotep Museum at Saqqara)
(Photograph courtesy of Alain Guilleux).

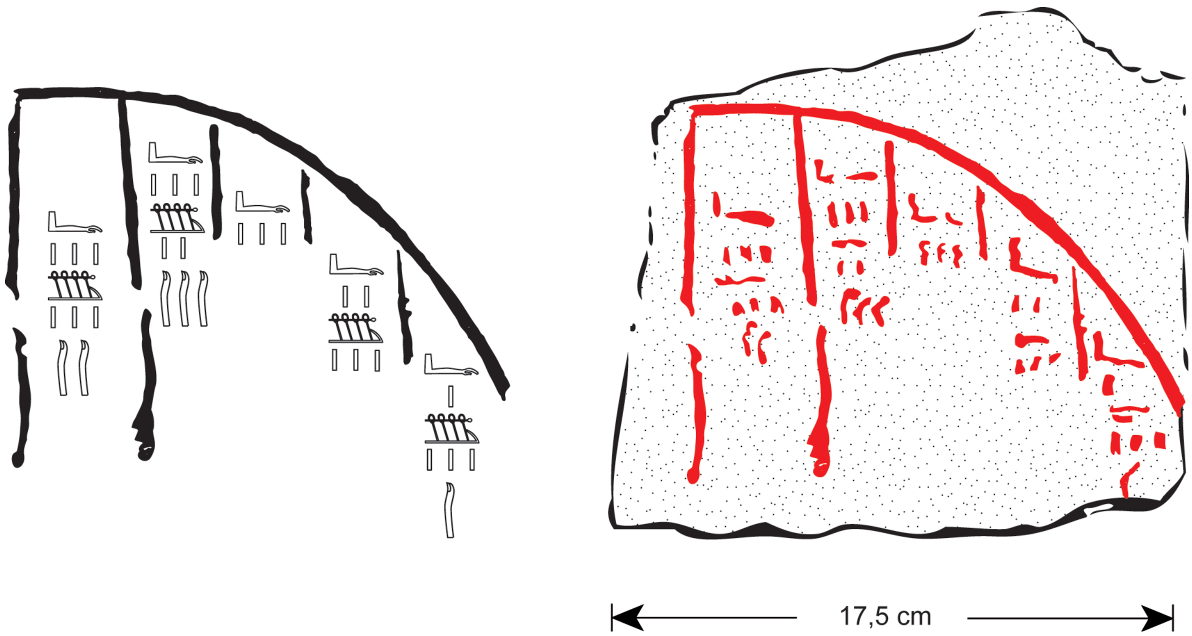

Fig. 5. Profile of a half vault or half arch represented on a limestone ostracon dating to the 3rd dynasty (JE 50036, previously in the Cairo Museum, now housed in the Imhotep Museum at Saqqara) and a hieroglyphic transcription of the annotations. The numbers inscribed in the object indicate that the sketch probably served as a construction guide for a vault, or arched decoration, and as such both sides would have been symmetrical so that only one side had to be depicted.

The curve drawn on it in red ink was accompanied by a number of inscriptions in hieratic which were regularly spaced by vertical lines (fig. 4). The drawing and the annotations beside it were addressed many decades ago by Battiscombe Gunn,[20] Georges Daressy,[21] Somers Clarke and Reginald Engelbach,[22] and subsequently by Jean-Philippe Lauer.[23] The numerals appearing on the diagram are expressed in cubits, palms, and digits, which Gunn reformulated into digits to facilitate mathematical analysis of the sequence of values found there (see table and figs. 5 and 6).

Translation of the dimensions listed	The same dimensions expressed only in digits
3 cubits, 3 palms, and 2 digits	98
3 cubits, 2 palms, and 3 digits	95
3 cubits	84
2 cubits, 3 palms	68
1 cubit, 3 palms, and 1 digit	41

Table. Dimensions recorded on ostracon JE 50036.

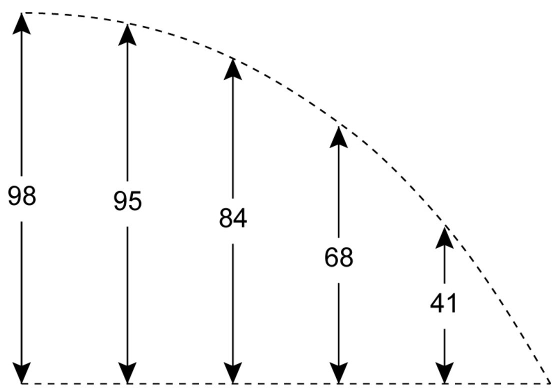

Fig. 6. Dimensions recorded on ostracon JE 50036.

Gunn interpreted the numbers as dimensions of a vault seen in profile.[24] The numerals would therefore indicate the height of the curve at the top of equidistant vertical 'ribs' or 'ordinates' located along its span.

The horizontal distance between the vertical values is not specified on the ostracon, but logic would suggest that they were separated by a constant value, which Gunn decided to set as 28 digits, or one cubit. He then sought to relate this design to the remains of a mound-like structure 'E' situated within the Djoser complex.[25] This relationship was rejected by Lauer based on later measurements.[26] In the present study, and in the absence of accurate data for the size of the structure

20 Gunn (1926).
21 Daressy (1927), pp. 157-160.
22 Clarke and Engelbach (1930), pp. 52-56.
23 Lauer (1936), pp. 174-175; Rossi (2003), pp. 115-117; El-Naggar (1999), pp. 331-332 described these artefacts simply by re-iterating the analyses and conclusions of their predecessors.
24 Gunn (1926), pp. 197-202.
25 Gunn (1926), pp. 200-202, fig. 5.
26 Lauer (1936), pp. 174-175, fig. 199.

'E', no attempt is made to compare the information on the ostracon with the monument, of which only a small part of the original material remains.

George Daressy re-opened the discussion shortly thereafter and provided his own analysis of the fragment.[27] He attempted to establish how the precise values recorded on the object were originally generated. Like Gunn he postulated that the values represented the heights of several points to be used to trace out the profile of the arc. He also concluded that the curve was a circular arc that had originally been traced out on the ground.

Daressy's diagram illustrating this method seems convincing, but when the current authors tried to verify its accuracy it was found that the arc did not match all of the recorded dimensions satisfactorily. In fact, according to the current authors' measurements the differences were not negligible, and this made us doubt that the ostracon represented a circular arc made in this way.

One of the main problems with Daressy's analysis is that he assumed that the gap between the vertical ribs at the far right hand side of the curve was in fact less than for the others. It is quite understandable if the scribe did not record the distances between each vertical rib because the distances were all effectively the same, but on the other hand, if one of the intervals were significantly less than for the rest then we would expect that this distance would have been recorded by necessity. In fact there is no such value recorded on the ostracon (at point 0 on the diagram). Daressy noted this discrepancy, but was content to explain it away by concluding that it was a detail which would have been known to the workers due to routine practices, and therefore did not need to be recorded.[28] However, this would only be the case if we assume that every vault constructed during that era (or perhaps just on this construction site) had an identical span, something that seems highly unlikely.

Both the circular arc proposed by Daressy and a catenary curve reveal small differences compared to the heights of the vertical ribs recorded on the ostracon.

Once again, the width of the vault was not the initial value used in Daressy's process, something that seems counter-intuitive. If the ancient Egyptian mason was dealing with a vault designed to span a space, then the width of the space was the initial dimension that had to be specified. In Daressy's method the width of the space is only derived at the end of the whole process.

Since a vault is designed to cover or protect a space it can only be designed and dimensioned once the space itself has been defined. It is, therefore, more likely that the vertical rises which defined the curve of the vault were derived from the horizontal base implied by the diagram, and in particular, from the width of this base.

We can now introduce and describe the new method which more accurately explains the data-set found on the ostracon from Saqqara. The method was first derived by chartered engineer Andrew Conner in 2004, and was subsequently published on his personal website which was devoted to the subject. Conner proposed that the arc described on the ostracon was half of a circular segment, but he suggested that the chord of the circular arc was based on a 3-4-5 triangle, for which the value 4 corresponded to half the width of the vault space to be covered, and the value 5 corresponded to the radius of the describing circle. The full width of the vault or arc would equate to a full chord of the circle.

27 Daressy (1927), pp. 157-160.
28 Daressy (1927), p. 158.

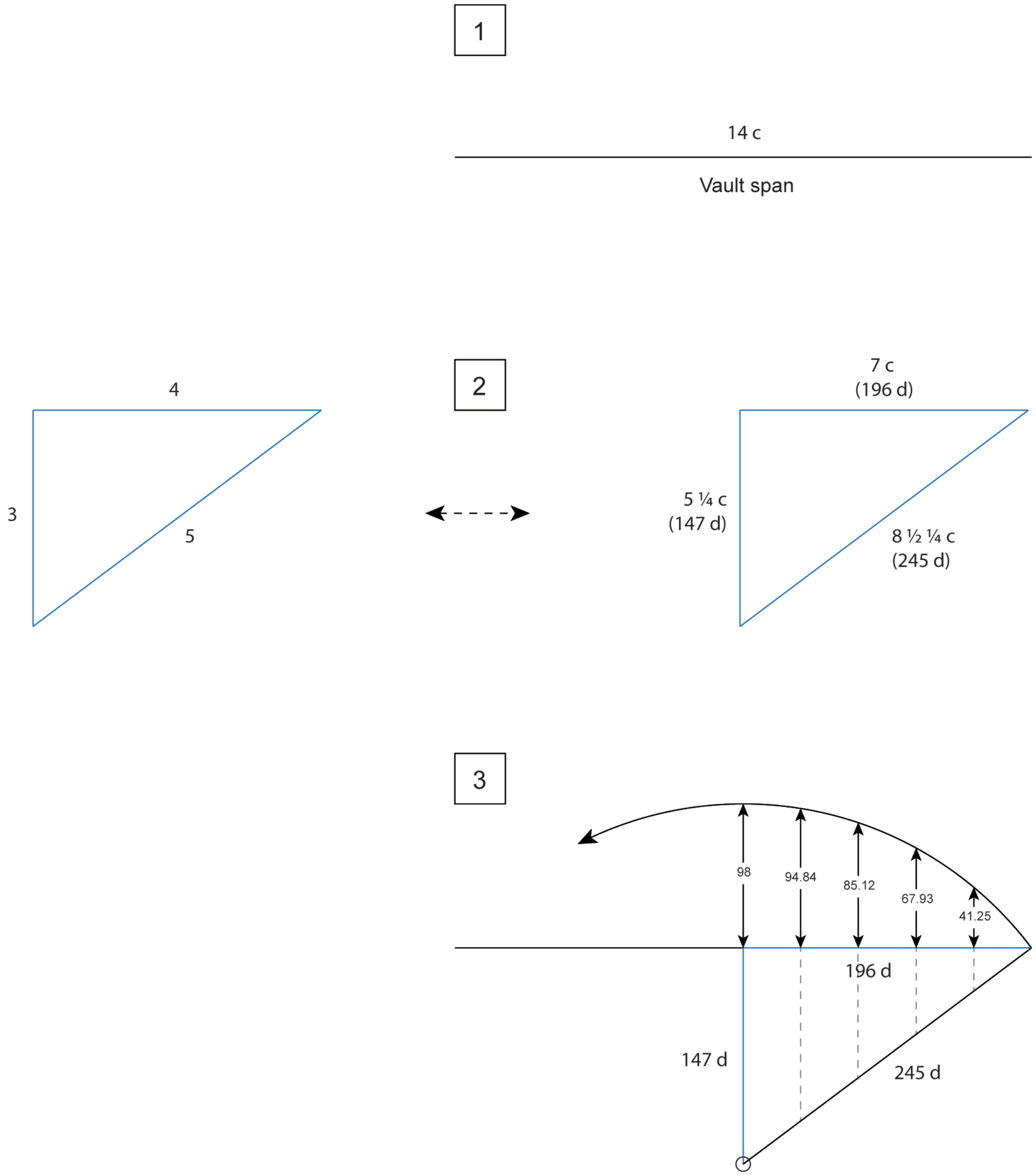

Fig. 7. Conner's method for obtaining the Saqqara Ostracon data.
(98, 95, 84/**85**, 68, 41, 0).

The strength of Conner's proposed method is that the curve described using this method matches the data described on the ostracon very closely, with only a single digit of discrepancy on one of the ordinates. Additionally, the width of the vault described would be a whole 14 cubits, something that agrees well with a theory where the width of the vault defined the dimensions of the vault's curvature.[29]

29 There is one apparent weakness to this theory in that the resulting distance between each of the five subdivisions (assuming they were all equal) is a non-integer value, i.e. 1.4 cubits. This distance is particularly difficult to measure out using a seven-part cubit. In fact, the 14 cubit width to be covered was particularly challenging in this respect, as its half-width is a prime number; seven cubits. This cannot be subdivided into a whole integer value, or a fractional value that can be easily measured using cubits and palms, unless it is subdivided into seven parts or more. One possibility is that the positions of the subdivisions were measured using the radius below as a guide, as it was constructed as 5 parts (in the 3-4-5 triangle) when making the curve (245/5=49 digits).

With respect to the one digit of discrepancy, Conner suggested that the third ordinate had flaked off or had accidently been omitted, in which case the value should be 85. The data produced by the geometric method would then provide an exact match to the data on the ostracon.

The method is simple and can be produced using basic practical instruments, readily available to the Egyptians of that era, and it does not require a background knowledge of more complex geometry.

Conner concluded that the method was equivalent to that used in constructing the data presented on the Saqqara ostracon, and he therefore proposed that the 3-4-5 triangle was known to the Old Kingdom Egyptians. The method is also identical to that proposed earlier in this article for the diagram depicted outside KV9.

Discussion

The curves studied in this article were most likely intended to guide the construction of curved vaults. Both examples describe arcs of similar magnitudes and almost identical proportions, and according to our analysis both were constructed based on 3-4-5 triangles. In the example from KV9 the vault has a width of 12 cubits and an internal height of 3 cubits, while in the second example on the ostracon from Saqqara the vault has a width of 14 cubits and an internal height of 3 ½ cubits. Both have the same form and proportions. The long side of the 3-4-5 triangle which determined those proportions would have been used as the radius of the describing circle. One of the strengths of this hypothesis is that the circular chords describe the width of the vaults. Chamber widths were the dimension of primary importance with respect to covering architectural spaces. Both spans are also whole numbers of cubits.

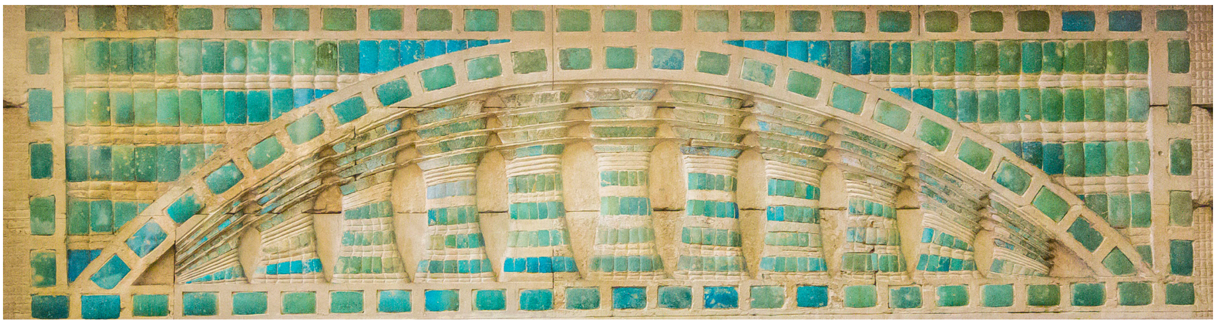

Fig. 8. Tiled faïence relief from the subterranean chambers beneath the Djoser Complex (JE 68921 Cairo Museum). (photo courtesy of Alain Guilleux)

This evidence indicates that the ancient Egyptians were already able to carry out relatively systematic and sophisticated geometric research and architectural construction during the 3rd dynasty. The 3rd and 4th dynasties were highly creative periods in pharaonic history. Many new construction techniques and fundamental new concepts first appeared at that time. In the decades during which the Djoser complex was built the scribes and architects attained unprecedented levels of technical ability, notable even within the wider context of the Old Kingdom. It is worth recalling that the architect Imhotep (c. 2650-2600 B.C.), who reputedly designed and constructed the Saqqara complex, was revered by later Egyptians for his exceptional skills and as a pioneer in building with stone.

During his research into the Saqqara ostracon, Conner also identified decorative features at Saqqara which are perhaps related to the vault structures and methods discussed here. Several faïence ceramic tiled reliefs discovered in the subterranean tunnels beneath the Saqqara step pyramid include designs incorporating circular segments, or arcs surmounting chords of circles.[30] Djed columns support the arcs over the chords, in positions comparable to those of the ordinates on the ostracon. In turn the arcs sit on large rectangles made up of many smaller rectangular ceramic tiles.[31] One of these reliefs, JE 68921, is now on display in the Cairo Museum. The close similarity between these decorative forms and the geometry described above leads us to consider if the ostracon is related to these reliefs in some more significant way. While the dimensions and proportions do not match in this case, it is worth considering if these reliefs resemble designs and elements that would have been familiar to the artisans and scribes of the period. If the ancient Egyptians were studying basic concepts of rectilinear and circular geometry, including arches and circular segments, at the time the Djoser complex was under construction, then it is possible that these tiles were based on elements that had a metrical function, and which could have facilitated experimental geometric research. Rather than being purely decorative, these reliefs may even have reused and preserved materials that also had a practical, and perhaps educational, function, or they may be durable skeuomorphs of more perishable wooden precursors.

The small tiles resemble what would result if a cubit were cut into pieces, for example if the parts were to be used to measure individually. Analysis of the dimensions of the tiles indicates that they were a fairly regular half-palm/2 digits in width (approximately 3.74 cm), so that 14 of them aligned side-by-side in a row would form a cubit.[32] Subdivision of rules into units of half-palms/2-digits would have been particularly useful for measurement. It would have allowed half cubits (3 ½ palms) and multiples of half cubits to be measured, something that could not be done using the basic 7-palm measure, or using individual whole palm units.

Setting out half-palm/2-digit tiles along circumferences of circles or arcs could have provided a simple means of measuring curved forms. If such a system existed, however, then it remains unclear from the reliefs if the tiles could be used to measure end-on-end, in the way that they are oriented around the curve on the relief (Fig. 8), or if that arrangement is purely a decorative form. The mortar separating the tiles in these arrangements also requires explanation if a metrical function was intended.

Despite being incapable of measuring half cubits accurately without further subdivision, the basic 7 part cubit was well suited to circular geometry. A circle with a 1 cubit diameter, being 7 palms, would have a circumference of precisely 22 palms. A quarter circle circumference would then be precisely 5 1/2 palms. These key values could easily have been measured by using small half-palm tiles placed around the circumferences of circles, perhaps set out on a horizontal surface.

Additional evidence from later Egyptian texts indicates possible continuity of these methods and concepts over longer periods of time. For example, demotic mathematical papyri from the Late Period include calculations involving circular segments and chords of circles.[33] The fundamental mathematical concepts were also understood by the neighboring complex ancient civilizations that

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30 Kuraszkiewicz (2015) discusses the significance of 64 different hieroglyphic marks recorded as inscribed on the reverse of the ceramic tiles. The marks include several numerals. It remains unclear what function these characters served, but Kuraszkewicz concluded that these were makers marks applied during the manufacturing process.

31 Sourouzian and Saleh (1986), p. 267; Lauer (1936), pp. 34-37. Excavations by the Egyptian Antiquities Service, 1928. The 'Panel with Mats Decoration from Faience Tiles' is a reconstruction of an unfinished wall found in the underground chambers. It is now on display in the Cairo Museum upper floor corridor 42. The height of the relief including the base rectangle is 181 cm, the width is 203 cm.

32 The width of the row of 20 upright tiles from the Djoser complex in Saqqara now in the NY Met (48.160.1) is 73.7 cm, giving an average width of 3.695 mm. This deviates by less than 1.2% from 2 digits, if the Old Kingdom cubit is taken as 0.5235 m (20.61"). A survey of several different tiles on sale for private collectors showed a width range varying from 36-38 mm.

33 See Parker (1972), pp. 44-50, plates 12-14, P. Cairo J.E. 89140, 89141, 89143.

developed in the Fertile Crescent. Cuneiform tablet MS3049, now[34] in Schoyen collection includes two diagrams and two examples demonstrating algorithms for calculating the lengths of chords of circles. The tablet was written in Old Babylonian, in cuneiform script on clay, and dates to the 17th century B.C.[35] Similarly, Euclid's Elements and Claudius Ptolemy's tables of chords later contained problems that demonstrated how to deal with circular chords.[36]

But why would the ancient Egyptians have employed a 3-4-5 triangle in this construction method? There are several plausible reasons. It generates a curve that can be produced consistently and in proportion for any given architectural span. It is an extremely strong form structurally, because the circular segment described is remarkably close in form to a catenary curve, which provides optimum structural strength. Lauer suggested that the 3-4-5 proportions were used in Egyptian architecture for reasons of architectural harmony of proportion,[37] but it is also possible that the form was originally chosen for the practical reasons noted above. It was inherently strong and could be generated quickly and precisely with rudimentary methods and tools. The ancient Egyptians may even have perceived that the 3-4-5 numerical sequence lent symbolic strength to the finished structure. Whatever the determining factors, the end result was an elegant and practical solution to a complex architectural problem.

Conclusion

In conclusion, the circular arc which has been described in this article was apparently used in two disparate architectural contexts separated by more than 1400 years. The characteristics of the curve and its applications are so noteworthy that it would be remarkable if they occurred by accident. The correspondence between the geometric form described and the numerical and dimensional data recovered is very close. The current authors consider that this special circular arc is very likely to be an archetypal geometric form, first devised, used, and then passed down by the ingenious scribes and engineers of Old Kingdom Egypt.

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34 Friberg (2007), pp. 295-304. The tablet was written in Old Babylonian cuneiform on clay and dates to the 17th century B.C. It was recovered from Uruk, now in modern day Iraq.

35 Friberg (2007), p. 298. See also Old Babylonian tablet BM85194, ##21-22 from Sippar.

36 Euclid, Prop. III.35, and Ptolemy's tables of chords 'On the Size of Chords Inscribed in a Circle', part of the Syntaxis Mathematica of the Almagest,from the 2nd century A.D.

37 Lauer (1968).

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