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MULTIDIMENSIONAL DISCRETE PDE SPLINES

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Abstract

Radial basis function (RBF) methods have emerged as an important and effective tool for the numerical solution of partial differential equations (PDE) in any number of dimensions and for the approximation of an unknown multivariate function by interpolation at scattered sites [2, 5], entering in a field traditionally tackled by finite element methods (FEM) [4].

Also, PDE surfaces, which are surfaces whose behaviour is governed by PDE [1], have been shown to possess many modelling advantages in a wide range of fields. A combination of conditions of interpolation and approximation can be used for the PDE method of surface design: on one hand, the surface has to approximate a given data set, and on the other hand, it has to be modelled by a partial differential equation. In addition, the surface has to satisfy some boundary conditions that are included along with the equation as a boundary value problem. Moreover, this 2-dimensional approximation problem may be generalized to the d-dimensional case, for any positive integer d.

We present the recent results of our work [3] in which, by using RBF techniques, we study the existence and the uniqueness of the solution of the generalized problem in a Lipschitz domain and arbitrary dimension. We discretize the solution of the problem in terms of RBF, we show the convergence of this solution to a function from which the data values are obtained and we establish some estimations of the error.

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