X Jaen Conference on Approximation
Úbeda, June 30th – July 5th, 2019

BEYOND SUPER-RESOLUTION

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Abstract

The problem of super-resolution in general terms is to recuperate a finitely supported measure μ given finitely many of its coefficients $\hat{\mu}(k)$ with respect to some orthonormal system. The interesting case concerns situations, where the number of coefficients required is substantially smaller than a power of the reciprocal of the minimal separation among the points in the support of μ.

In this paper, we consider the more severe problem of recuperating μ approximately without any assumption on μ beyond having a finite total variation. In particular, μ may be supported on a continuum, so that the minimal separation among the points in the support of μ is 0. A variant of this problem is also of interest in machine learning as well as the inverse problem of de-convolution.

We define an appropriate notion of a distance between the target measure and its recuperated version, give an explicit expression for the recuperation operator, and estimate the distance between μ and its approximation. We show that these estimates are the best possible in many different ways. We also explain why for a finitely supported measure the approximation quality of its recuperation is bounded from below if the amount of information is smaller than what is demanded in the super-resolution problem.

Keywords: Super-resolution, machine learning, de-convolution, data defined spaces, widths

AMS Classification: 41A30, 28A33

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