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METRIC APPROXIMATION OF SET-VALUED FUNCTIONS OF BOUNDED VARIATION

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Abstract

We study approximation of set-valued functions (SVFs) — functions mapping a real interval to compact sets in $\mathbb{R}^d$. In addition to the theoretical interest in this subject, it is relevant to various applications in fields where SVFs are used, such as economy, optimization, dynamical systems, geometric modeling.

The earlier works in this area are mainly concerned with approximation of set-valued functions with convex images, for which the tools of Minkowski linear combinations of sets and the Aumann integral [1] of set-valued functions are effective. Yet these techniques posses the property of convexification: the resulting approximation is always a function with convex images, even if the function to be approximated is not. For example, R.A. Vitale [4] studied an adaptation of the classical Bernstein polynomial operator based on Minkowski linear combinations; the limit SVF in this approach consists of convex hulls of the values of the original function. Clearly, such methods are useless for the approximation of SFVs with general, not necessarily convex images.

N. Dyn, E. Farkhi and A. Mokhov developed a new approach that is free of convexification — the so-called metric linear combinations and the metric integral, see [3]. They introduced and studied adaptations of classical approximation operators based on these tools for continuous SFVs.

The next step is to study SFVs that are not necessarily continuous. In our joint work we consider SVFs of bounded variation in the Hausdorff metric and adopt different types of classical approximation operators to this setting. Some of the results can be found in [2].

Keywords: set-valued functions, metric linear combinations, metric integral, Bernstein operator.

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