X Jaen Conference on Approximation
Úbeda, June 30th – July 5th, 2019

ON COHERENT PAIRS OF POLYNOMIAL SYSTEMS IN TWO VARIABLES

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Abstract

Coherent pairs of measures were introduced in 1991 and constitute a very useful tool in the study of Sobolev orthogonal polynomials on the real line. In this work, coherence and partial coherence in two variables appear as the natural extension of the univariate case. Given two families of bivariate orthogonal polynomials expressed as *polynomial systems*, they are a *partial coherent pair* if there exists a polynomials of the second family can be given as a linear combination of the first partial derivatives of (at most) three consecutive polynomials of the first family. A *full coherent pair* is a pair of families of bivariate orthogonal polynomials related by means of partial coherent relations in each variable. Consequences of this kind of relations concerning both families of bivariate orthogonal polynomials are studied.

Keywords: Bivariate orthogonal polynomials, classical and semiclassical orthogonal polynomials, coherent pairs.

Bibliography

[1] C. F. Dunkl, Y. Xu, *Orthogonal polynomials of several variables*, 2nd edition, Encyclopedia of Mathematics and its Applications, vol. **155**, Cambridge Univ. Press, Cambridge, 2014.

[2] A. Iserles, P. E. Koch, S. P. Nørsett, J. M. Sanz-Serna, *On polynomials orthogonal with respect to certain Sobolev inner products*, J. Approx. Theory **65** (1991), 151-175.

[3] H. L. Krall, I. M. Sheffer, *Orthogonal polynomials in two variables*, Ann. Mat. Pura Appl. (4) **76** (1967), 325-376.

The research of F. Marcellán and M. E. Marriaga is supported by Ministerio de Economía, Industria y Competitividad of Spain, grant MTM2015-65888-C4-2-P. The research of T. E. Pérez and M. A. Piñar is supported by Ministerio de Economía, Industria y Competitividad of Spain and the European Regional Development Fund (ERDF) the the grant MTM2014-53171-P, and Junta de Andalucía gran P11-FQM-7276 and Research Group FQM-384.

[4] F. Marcellán, A. Branquinho, and J. Petronilho, *Classical orthogonal polynomials: a functional approach*, Acta Appl. Math. **34** (1994), 283-303.

[5] F. Marcellán, M. E. Marriaga, T. E. Pérez, M. A. Piñar, *On bivariate classical orthogonal polynomials*, Appl. Math. Comp. **325** (2018), 340–357.

[6] F. Marcellán, M. E. Marriaga, T. E. Pérez, M. A. Piñar, *Matrix Pearson equations satisfied by Koornwinder weights in two variables*, Acta Appl. Math. **153** (2018), 81–100.

[7] M. E. Marriaga, T. E. Pérez, M. A. Piñar, *Three term relations for a class of bivariate orthogonal polynomials*, Mediterr. J. Math. **14** (2017), no. 2, Art. 54, 26 pp.

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