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POLYNOMIAL APPROXIMATION ON C^2 DOMAINS

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Abstract

We introduce appropriate computable moduli of smoothness to characterize the rate of best approximation by multivariate polynomials on a connected and compact C^2-domain $\Omega \subset \mathbb{R}^d$. This new modulus of smoothness is defined via finite differences along the directions of coordinate axes, and along a number of tangential directions from the boundary. With this modulus, we prove both the direct Jackson inequality and the corresponding inverse for best polynomial approximation in $L_p(\Omega)$. The Jackson inequality is established for the full range of $0 < p \leq \infty$, while its proof relies on (i) Whitney type estimates with constants depending only on certain parameters; and (ii) highly localized polynomial partitions of unity on a C^2-domain. Both (i) and (ii) are of independent interest. In particular, our Whitney type estimate (i) is established for directional moduli of smoothness rather than the ordinary moduli of smoothness, and is applicable to functions on a very wide class of domains (not necessarily convex). It generalizes an earlier result of Dekel and Leviatan on Whitney type estimates on convex domains. The inverse inequality is established for $1 \leq p \leq \infty$, and its proof relies on a new Bernstein type inequality associated with the tangential derivatives on the boundary of Ω. Such an inequality also allows us to establish the Marcinkiewicz-Zygmund type inequalities, positive cubature formula, as well as the inverse theorem for Ivanov's averaged moduli of smoothness on general compact C^2-domains.

Keywords: C^2-domains, polynomial approximation, modulus of smoothness, Jackson inequality, inverse theorem, tangential Bernstein inequality, Marcinkiewicz-Zygmund inequality, positive cubature formula.

AMS Classification: 54C40, 14E20, 46E25, 20C20.

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