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A class of orthogonal polynomials on the unit circle and related special functions

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Abstract

The class of orthogonal polynomials $\Phi_n(b; \cdot)$ that we consider are those with respect to the probability measures

$$d\mu^{(b)}(e^{i\theta}) = \frac{4^{\Re(b)} |\Gamma(b+1)|^2}{\Gamma(b+\bar{b}+1)} \frac{1}{2\pi} [e^{\pi-\theta}]^{\Im(b)} [\sin^2(\theta/2)]^{\Re(b)} d\theta,$$

on the unit circle, where $b = \lambda + i\eta$ and $\lambda > -1/2$. We will give information about a sequence $\{R_n(b; \cdot)\}_{n \geq 0}$ of para-orthogonal polynomials and the associated sequence of polynomials $\{P_n(b; \cdot)\}_{n \geq 0}$ which satisfy a R_{II} type recurrence relation. All three sets of polynomials $\Phi_n(b; \cdot)$, $R_n(b; \cdot)$ and $P_n(b; \cdot)$ can be expressed as hypergeometric polynomials. Generating functions for all these sets of polynomials are also considered, which in the case of $P_n(b; \cdot)$, lead to a subfamily of Whittaker functions. This subfamily of Whittaker functions also include Coulomb wave functions and Bessel functions. Finally, properties of associated Sobolev orthogonal polynomials, based on a concept which we have called *coherent pair of measures of the second kind on the unit circle*, are also looked at. Some sequences of polynomials that arise in this study are found to be related to continuous dual Hahn polynomials.

Bibliography

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