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- Jesús Carnicer,
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- Say Song Goh,
  National University of Singapore,
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- Gitta Kutyniok,
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Introduction

This is the eighth edition of the Jaen Conference on Approximation Theory, an activity within the Jaen Approximation Project. This project organized ten editions of the so-called Ubeda Meeting for ten consecutive years, from 2000 to 2009, and nowadays issues the Jaen Journal on Approximation. This periodical, launched in 2009, was recently invited to be indexed in Emerging Sources Citation Index, Thomson Reuters.

The Conference is devoted to some significant aspects on Approximation Theory, Computer Aided Geometric Design, Numerical Methods and the Applications of these fields in other areas. The main objective is to provide a useful and nice forum for researchers in the subjects to meet and discuss. In this sense, the conference program has been designed to keep joined the group during four days taking special care of both scientific and social activities.

The Conference features six invited speakers who will give plenary lectures: Feng Dai, Gitta Kutyniok, Jesús Carnicer, Martin Muldoon, Say Song Goh and Vladimir Temlyakov. About twenty short talks and a poster session have been scheduled as well.

Finally, but also important, the Conference provides to participants the possibility to visit World Heritage Sites and taste a wide culinary variety. We will do all the best for accompanying people to enjoy the Conference. We are grateful to all those who have made this project a reality; the University of Jaén (Vicerrectorado de Investigación, Departamento de Matemáticas), Diputación Provincial de Jaén, Ayuntamiento de Úbeda, UNIA (Sede de Baeza) and Centro Asociado de la UNED de la provincia de Jaén.

Here we emphasize our commitment to keep on working to improve our university and our province. The Organizing Committee

The Organizing Committee

Scientific Program

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21:00-	**Dinner** (Hotel María de Molina)

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Invited Lectures

Maximal lines of sets satisfying the geometric characterization*

J. M. Carnicer and C. Godés

Abstract

The geometric characterization introduced by Chung and Yao [3] identifies unisolvent sets for total degree interpolation such that their Lagrange polynomials are products of linear factors. Sets satisfying the geometric characterization for degree n are usually called GC_n sets. Gasca and Maeztu [4] conjectured that planar GC_n sets contain $n+1$ collinear points and hence GC_n sets are particular instances of Berzolari-Radon sets [1, 6]. Maximal lines, introduced by C. de Boor [2] to analyze the problem, are lines containing $n+1$ nodes of a unisolvent set of degree n. In recent papers [5], the conjecture has been revisited showing that it holds for degrees $n \leq 5$. Unfortunately, the conjecture is still unsolved for general degree. In the solution of the conjecture new lines of research have been proposed, such as using common tools in algebraic geometry. One promising approach consists in studying the relations between the generators of the ideal of polynomials vanishing at the nodes. We propose an analysis of the extension of a GC_n set to a GC_{n+1} set by adding a $n+1$ nodes on a line as a tool to deepen in the structure on GC_n sets and its relationship with the Berzolari-Radon construction.

Keywords: geometric characterization, Gasca-Maeztu conjecture, maximal lines.
AMS Classification: 41A05, 41A63, 65D05.

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• VIII Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 2nd-7th, 2017

Chebyshev-type cubature formulas for doubling weights on spheres, balls and simplexes*

Feng Dai and Han Feng

Abstract

In this talk, I will report my recent joint work with Han Feng on strict Chebyshev-type cubature formulas (CF) (i.e., equal weighted CFs) for doubling weights w on the unit sphere $\mathbb{S}^{d-1}$ of $\mathbb{R}^d$ equipped with the usual surface Lebesgue measure $d\sigma_d$ and geodesic distance $\mathrm{dist}(\cdot,\cdot)$. Our main interest is on minimal number $\mathcal{N}_n(wd\sigma_d)$ of nodes required in a strict Chebyshev-type CF of degree n for a doubling measure $wd\sigma_d$ on $\mathbb{S}^{d-1}$. One of our main results states that for a doubling weight w on $\mathbb{S}^{d-1}$,

$$\mathcal{N}_n(wd\sigma_d) \sim \mu_{n,w} := \max_{x\in\mathbb{S}^{d-1}} \frac{1}{w(B(x,n^{-1}))},$$

where the constants of equivalence are independent of n, and $B(x,r)$ denotes the spherical cap with center $x \in \mathbb{S}^{d-1}$ and radius $r > 0$. In fact, we will prove that given a normalized doubling weight w on $\mathbb{S}^{d-1}$, there exists a positive constant K_w depending only on the doubling constant of w such that for each positive integer n and each integer $N \geq K_w\mu_{n,w}$, there exists a set of N distinct nodes $z_1, \cdots, z_N$ on $\mathbb{S}^{d-1}$ which admits a strict Chebyshev-type cubature formula (CF) of degree n for the measure $w(x)d\sigma_d(x)$, and which satisfies

$$\min_{1\leq i\neq j\leq N} \mathrm{dist}(z_i, z_j) \geq c_* N^{-\frac{1}{d-1}}$$

if in addition $w \in L^\infty(\mathbb{S}^{d-1})$. The proofs of these results rely on new convex partitions of $\mathbb{S}^{d-1}$ that are regular with respect to the weight w. The weighted results on the sphere also allow us to establish similar results on strict Chebyshev-type CFs on the unit ball and the standard simplex of $\mathbb{R}^d$.

Our results generalize the recent results of Bondarenko, Radchenko, and Viazovska on spherical designs.

Keywords: Chebyshev-type cubature formulas for doubling weights, spherical designs, spherical harmonics, convex partitions of the unit spheres.

*This work was supported by NSERC Canada under grant RGPIN 04702.

AMS Classification: 41A55, 41A63, 52C17, 52C99, 65D32.

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Frames on locally compact abelian groups

Say Song Goh

Abstract

Gabor frames and wavelet frames for $L^2(\mathbb{R})$ are redundant systems which facilitate sparse representations of signals and images, and they play important roles in many practical applications. We shall present a unifying generalization of these frames to locally compact abelian groups. This generalization, in the notion of Fourier-type frames, covers both the stationary and nonstationary case, as well as various variants of Gabor frames and wavelet frames in the literature. Our focus is on the development of useful methods for explicit constructions of Fourier-type frames, including the unitary extension principle on locally compact abelian groups. The resulting Fourier-type frames, defined on the dual group, are generated by modulates of a collection of functions, which correspond, via the Fourier transform, to generalized shift-invariant systems on the group. We shall also introduce weighted B-splines on locally compact abelian groups, which are used to construct localized Gabor frames on the dual group and localized tight wavelet frames on the group. This is joint work with Ole Christensen.

Keywords: Gabor frames, wavelet frames, weighted B-splines, locally compact abelian groups.

AMS Classification: 41A15, 42C15, 43A70.

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Approximation theory meets deep learning

Gitta Kutyniok

Abstract

Despite the outstanding success of deep neural networks in real-world applications, most of the related research is empirically driven and a mathematical foundation is almost completely missing. One central task of a neural network is to approximate a function, which for instance encodes a classification task. In this talk, we will be concerned with the question, how well a function can be approximated by a neural network with sparse connectivity. Using methods from approximation theory and applied harmonic analysis, we will derive a fundamental lower bound on the sparsity of a neural network. By explicitly constructing neural networks based on certain representation systems, so-called α-shearlets, we will then demonstrate that this lower bound can in fact be attained. Finally, we present numerical experiments, which surprisingly show that already the standard backpropagation algorithm generates deep neural networks obeying those optimal approximation rates.

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Results and conjectures on zeros of special functions*

Martin E. Muldoon

Abstract

I will describe some highlights and some occasional unsolved problems related to work that I have done over the past 50 years, with a variety of authors including Lee Lorch, Peter Szego, John T. Lewis, Mourad Ismail, Andrea Laforgia, Panos D. Siafarikas, Árpad Elbert, Dharma P. Gupta and, most recently, Kathy Driver.

Questions that arise include the reality of the zeros, and how they vary with parameters, including monotonicity, convexity and higher monotonicity properties. Other questions relate to inequalities, asymptotic properties, and interlacing properties.

Keywords: Bessel functions, orthogonal polynomials, zeros, inequalities, interlacing, parameter dependence, approximation.

AMS Classification: 26C10, 33C10, 33C45, 34C10.

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The Marcinkiewicz-type discretization theorems for the hyperbolic cross polynomials*

Vladimir Temlyakov

Abstract

The talk is devoted to discretization of integral norms of functions from a given finite dimensional subspace – the hyperbolic cross polynomials. This problem is important in applications but there is no systematic study of it. We present here a new technique, which works well for discretization of the integral norm. It is a combination of probabilistic technique, based on chaining, with results on the entropy numbers in the uniform norm.

Keywords: discretization, entropy numbers, sparse approximation, chaining technique.
AMS Classification: 41A60, 42A10, 46E35.

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Short Talks/Posters

Some new fixed point results for ordered F-contractions

Özlem Acar

Abstract

In this talk, we mainly study on fixed point theorem for ordered multivalued mappings with δ-distance using Wardowski's technique on complete metric space. Considering δ-distance, we proof some fixed point results and give some corollary.

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Some recent results for pointwise convergence of linear positive operators*

Tuncer Acar and Ali Aral

Abstract

In the present talk, we present some recent results for pointwise convergence of linear positive operators in weighted spaces. The results consist of quantitative Voronovskaya type theorems for the family of operators acting on unbounded intervals.

Keywords: linear positive operators, Voronovskaya theorem, weighted spaces
AMS Classification: 41A28, 41A40, 41A60.

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Linking of Bernstein-Chlodowsky and Szász-Appell-Kantorovich type operators

P.N. Agrawal, Dharmendra Kumar and Serkan Araci

Abstract

In the present paper we define a sequence of bivariate operators by linking the Bernstein-Chlodowsky operators and the Szász-Kantorovich operators based on Appell polynomials. First, we establish the moments of the operators and then determine the rate of convergence of these operators in terms of the total and partial modulus of continuity. Next, we obtain the order of approximation of the considered operators in a weighted space . Furthermore, we define the associated GBS(Generalized Boolean Sum) operators of the linking operators and then study the rate of convergence with the aid of the Lipschitz class of Bögel continuous functions and the mixed modulus of smoothness.

Keywords: Appell polynomials, weighted approximation, GBS operators, partial and mixed modulus of smoothness, Peetre's K-functional.

AMS Classification: 41A10, 41A25, 41A36.

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• VIII Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 2nd-7th, 2017

Popoviviu-Ionescu functional equation revisited

J. M. Almira

Abstract

We study the functional equation

$$\det \begin{bmatrix} f(x) & f(x+h) & \cdots & f(x+nh) \\ f(x+h) & f(x+2h) & \cdots & f(x+(n+1)h) \\ \vdots & \vdots & \ddots & \vdots \\ f(x+nh) & f(x+(n+1)h) & \cdots & f(x+2nh) \end{bmatrix} = 0,$$

which was first proposed by T. Popoviciu [6] in 1955. It was solved for the easiest case by Ionescu [4] in 1956 and, for the general case, by Ghiorcoiasiu and Roscau [5] and Radó [7] in 1962. Our solution is based on a generalization of Radó's theorem to distributions in a higher dimensional setting and, as far as we know, is different than existing solutions. Finally, we propose several related open problems.

Keywords: functional equations, exponential polynomials on Abelian groups, Montel type theorem.
AMS Classification: 39B22, 39A70, 39B52.

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• VIII Jaen Conference on Approximation Theory
• Úbeda, Jaén, Spain, July 2nd-7th, 2017

Generalized Kantorovich operators and their associated positive semigroups

Francesco Altomare, Mirella Cappelletti Montano, Vita Leonessa and Ioan Raşa

Abstract

Deepening the study of an approximation sequence of positive linear operators which was introduced and studied in [1], in the paper [2] the authors investigated its relationship with the semigroup (pre)generation problem for a class of degenerate second-order elliptic differential operators of the form

$$A(u)(x) = \frac{1}{2}\sum_{i,j=1}^{d} \alpha_{ij}(x)\frac{\partial^2 u}{\partial x_i \partial x_j}(x) + \sum_{i=1}^{d} a(b_i - x_i)\frac{\partial u}{\partial x_i}(x)$$

($u \in C^2(K), a \geq 0, b = (b_1, \ldots, b_d) \in K, x = (x_1, \ldots, x_d) \in K$), where K is an arbitrary compact convex subset of $\mathbb{R}^d$, $d \geq 1$, having non-empty interior and a not necessarily smooth boundary.

In particular, they showed that the generated Markov semigroup, i.e., the solutions of the initial-boundary value problems

$$\begin{cases} \dfrac{\partial u}{\partial t}(x,t) = A(u(\cdot,t))(x) & x \in K, \quad t \geq 0; \\ \\ u(x,0) = u_0(x) & u_0 \in D(A), \quad x \in K. \end{cases} \tag{1}$$

can be approximated in terms of iterates of such approximating linear positive operators.

The talk is devoted to present some of the main results of [2]. The analysis is carried out in the context of the space $C(K)$ of all continuous functions defined on K as well as, in some particular cases, in $L^p(K)$ spaces, $1 \leq p < +\infty$. The approximation results also allow to infer some preservation properties of the semigroup such as the preservation of the Lipschitz-continuity as well as of the convexity which, in turn, highlight some spatial regularity properties of the solutions u of (1), i.e., regularity properties of the functions $u(\cdot,t)$, $t \geq 0$.

The main results are finally applied to some noteworthy particular settings such as the unit interval and the multidimensional balls, ellipsoids, hypercubes and simplices. In these

settings the relevant differential operators fall into the class of Fleming-Viot operators which appear in the description of stochastic processes associated with some gene frequency models in population genetics.

Keywords: positive approximation process, Kantorovich operator, positive semigroup, approximation of semigroup, degenerate second-order elliptic differential operator, Fleming-Viot operator.

AMS Classification: 47D07, 47B65, 35K65, 41A36, 41A63.

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Quantiative type theorems

Ali Aral

Abstract

We present some old and new quantitative results. Firstly we present quantitative Voronovskaya-type results and Grüss–Voronovskaya inequalities for polynomial bounded functions. Then we give quantitative Voronovskaya type theorems for the first and second derivative of general linear positive operators in weighted spaces. Finally, quantitative type results are given by estimating magnitude of differences of positive linear operators.

All results hold for a large class of linear positive operators defined unbounded intervals. Also, our measuring tools will be weighted K-functionals and different type modulus of continuity.

Keywords: quantiative approximation, Voronovskaya type theorems, weighted approximation.

AMS Classification: 41A36, 41A25.

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Generalized Voronovskaja-type quantitative estimates*

José A. Adell and Daniel Cárdenas-Morales

Abstract

In this talk we give new upper bounds for the uniform central moments of even order of the Bernstein polynomials. As a consequence, we present generalized Voronovskaja's formulae in quantitative form. Special interest is focused on the so called Videnskiĭ inequality.

Keywords: Bernstein polynomials, modulus of continuity, quantitative Voronovskaja formula, central moments.
AMS Classification: 41A25, 41A36, 41A44.

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On a robust iterative method for solving nonlinear equations*

Ion Păvăloiu and Emil Cătinaş

Abstract

We present an Aitken-Newton iterative method of Steffensen type for solving nonlinear equations, which is obtained by using the Hermite inverse interpolation polynomial.

A local convergence result is shown, which implies that the convergence order of the iterates is 8.

We also prove that under some supplementary conditions the iterations converge monotonically to the solution. This approach constitutes an alternative to the usual estimation of the radius of attraction balls in ensuring the convergence of the iterates.

Numerical examples show that this method may become competitive and in certain circumstances even more robust than certain optimal methods of same convergence order.

Keywords: nonlinear equations in $\mathbb{R}$, Aitken-Newton iterative methods, monotone convergence.

AMS Classification: 65H05.

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On some interpolation operators on triangles with curved sides

Teodora Cătinaş

Abstract

We present some results regarding interpolation operators and linear, positive operators defined on triangles having one curved side. They are extensions of the corresponding operators for functions defined on triangles with straight sides.

The operators defined on domains with curved sides permit essential boundary conditions to be satisfied exactly and they have important applications in: finite element methods for differential equations with given boundary conditions, the piecewise generation of surfaces in CAGD, in obtaining Bezier curves/surfaces in CAGD and in construction of surfaces that satisfy some given conditions.

We consider some Lagrange, Hermite and Birkhoff type interpolation operators and Bernstein and Cheney-Sharma type operators on triangles with one curved side. We construct their product and Boolean sum and study their interpolation properties.

We study three main aspects of the constructed operators: the interpolation properties, the orders of accuracy and the remainders of the corresponding interpolation formulas.

We use some of the interpolation operators and some of the Bernstein type operators for construction of surfaces that satisfy some given conditions, such as the roofs of the halls.

Finally, we give some numerical examples and we study the approximation errors for the operators presented here.

Keywords: triangle with one curved side, interpolation operators, orders of accuracy, remainders.

AMS Classification: 41A05, 41A25, 41A80.

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A new quasi wavelet method for multi-resolution analysis of 3D objects

Haroun Djaghloul, Abdelhamid Benhocine and Jean-Pierre Jessel

Abstract

During the past few years, wavelets have tremendously gain in popularity thanks to their wide range of applications due to their particular and exceptional properties overcoming by far other analysis and synthesis techniques. In particular, in image analysis and processing, wavelets can be used at different processing levels. In this study, we present a new lazy method to perform multi-resolution and represntation of 3D objects that can be represented using various techniques going from implicit to explicit mesh and voxels and point clouds. In particular, we present new analysis and synthesis filters. This wavelet family is characterized by two specific filters, namely, analysis and synthesis filters. These filters have been proven algebraically. The proosed quasi-wavelet family can perform at various dimensions and ,thus, with different datasets scopes and ranges. The wavelet can be applied for the multiresolution analysis of multi-dimensional discrete datasets especially 3D modelled objects using various techniques such as mesh and voxels based models. Several applications can be found such as compression, watermarking and multi-resolution analsyis and representation.

Keywords: lazy wavelet, multi-resolution analysis, 3D object.
AMS Classification: 41Axx, 42Axx, 42Cxx, 43Axx, 46Cxx, 47Axx, 94Axx.

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Zeros of ultraspherical and pseudo-ultraspherical polynomials

Kathy Driver

Abstract

The pseudo-ultraspherical polynomial of degree n is defined by $\tilde{C}_n^{(\lambda)}(x) = (-i)^n C_n^{(\lambda)}(ix)$ where $C_n^{(\lambda)}(x)$ is the ultraspherical polynomial. We discuss the orthogonality of finite sequences of pseudo-ultraspherical polynomials $\{\tilde{C}_n^{(\lambda)}\}_{n=0}^N$ for different values of N that depend on λ. We discuss applications of Wendroff's Theorem and use an identity linking the zeros of the pseudo-ultraspherical polynomial $\tilde{C}_n^{(\lambda)}$ with the zeros of the ultraspherical polynomial $C_n^{(\lambda')}$ where $\lambda' = \frac{1}{2} - \lambda - n$ to prove that when $1 - n < \lambda < 2 - n$, two (symmetric) zeros of $\tilde{C}_n^{(\lambda)}$ lie on the imaginary axis.

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Approximation problems in ECG signal processing*

Sándor Fridli

Abstract

The so called transformation method turned to be very effective in several areas of signal processing. Besides the classical trigonometric Fourier, polynomial transforms many other transforms like those based on wavelets have been used in various applications. We will concentrate on problems that are raised in ECG signal processing. Such problems include filtering, representation, compression of the signal, segmentation and classification of heartbeats. We show that the rational orthogonal systems are especially efficient in these areas. Namely, the algorithms based on them outperform the previous ones in many respects. We note that these systems have free parameters that can be adjusted to the individual problem. We consider, among others, issues like optimization, biorthogonality and discretization.

Keywords: signal processing, variable projection, rational systems.
AMS Classification: 41A20.

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Generalized convergence and approximation theory*

P. Garrancho

Abstract

Some notions of generalized convergence have been treated in Approximation Theory by mean of linear operators. Qualitative results, quantitative results, asymtpotic condition and saturation results were studied. Here, the author give a new notion of generalized convergence, the B-statistical $\tilde{\mathcal{A}}$-summability. This new notion generalizes, for example, the almost convergence. Some results about the usual topics are stablished. Finally, several conditions about generalized convergence are presented with the purpose of transferring them to the Approximation Theory.

Keywords: statistical-summability, simultaneous approximation, saturation.
AMS Classification: 41A28, 41A40, 41A60.

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Approximation by certain hybrid operators

Vijay Gupta

Abstract

Here we discuss approximation properties of some summation-integral type operators. We obtain the moments by using different methods for such operators and study some direct approximation results in ordinary and simultaneous approximation.

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On a family of exponential type operators

Monika Herzog

Abstract

In 1978 Mourad E. H. Ismail and C. Ping May investigated an exponential operator

$$S_\lambda(f,t) = \int_{\mathbb{R}} W(\lambda,t,u) f(u) du$$

with the normalization condition

$$\int_{\mathbb{R}} W(\lambda,t,u) du = 1,$$

where W - the kernel of S_λ is a positive function satisfying the following homogenous partial differential equation

$$\frac{\partial W}{\partial t}(\lambda,t,u) = \frac{\lambda}{p(t)} W(\lambda,t,u)(u-t),$$

$p = p(t)$ is analytic and positive for $t \in (A,B)$ with some $-\infty \le A < B \le +\infty$ and $\lambda > 0$.

For example, the Bernstein polynomials and opertators of Szász-Mirakjan, Post-Widder, Gauss-Weierstrass and Baskakov are exponential type operators. It is worth noting that all the above mentioned operators are approximation operators. Moreover, they satisfy the condition $S_\lambda(e_1,t) = e_1(t)$ where $e_1(t) = t$ for $t \in (A,B)$.

In 2005 A. Tyliba and E. Wachnicki extended the results of May and Ismail to a family of operators S_λ such that the condition $S_\lambda(e_1,t) = e_1(t)$ is not fulfilled. In this case, instead of the previous differential equation, they consider the following

$$\frac{\partial W}{\partial t}(\lambda,t,u) = \frac{\lambda}{p(t)} W(\lambda,t,u)(u-t) - \beta W(\lambda,t,u),$$

where β is a non-negative real number, $\lambda > 0$ and $u, t \in (A,B)$.

Our purpose is to study exponential type operators S_λ satisfying the differential equation considered in the paper of May, Ismail and the condition $S_\lambda(e_2,t) = e_2(t)$, where $e_2(t) = t^2$ for $t \in (A,B)$.

Keywords: exponential operators, modified Bessel function, rate of convergence.
AMS Classification: 41A25, 41A36.

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Sign sensitive weighted rational approximation

Miguel A. Jiménez Pozo and José N. Méndez-Alcocer

Abstract

It is considered the algebraic rational asymmetric approximation of continuous real valued functions on a compact real interval. One of the most relevant problem in this setting is the explicit calculation of elements of best approximation. But it is known that even for the uniform rational approximation the Rémez algorithm fails. Since possibilities of its convergence increase substantially whenever the function to be approximated is normal, we extend and study this concept for the asymmetric case. It is proved by examples that the normality condition of a function strongly depends on the sign sensitive weight, but the main known properties of normal functions in the uniform case still remain to be true in the asymmetric situation.

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The mixed finite elements for Navier-Lame problem

Ouadie Koubaiti, Jaouad El-Mekkaoui and Ahmed Elkhalfi

Abstract

In this article we solve the Navier-Lame problem in 2D with Dirichlet and Neumann boundary, using the Mixed Finite Element P1-bubble P1.We introduce a new weak formulation of this problem with help of another unknown equal to divergence of the displacement.We do the necessary calculations of this problem in order to imply a Matlab program that visualizes the numerical solution. Some numerical results are shown,prove that our method is more efficient than the ordinary Finite Element.

Keywords: Navier-Lame, mixed finite elements.
AMS Classification: 74S05, 78M10, 80M10.

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Schur type inequalities for multivariate polynomials on convex bodies*

András Kroó

Abstract

In this talk we give sharp Schur type inequalities for multivariate polynomials with generalized Jacobi weights on arbitrary convex domains. In particular, these results yield estimates for norms of factors of multivariate polynomials.

Keywords: convex bodies, multivariate polynomials, Schur type inequalities, Jacobi weights, doubling weights, zero index

AMS Classification: 41A17, 41A63.

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Asymptotic expansion and localization results for Taylor-Durrmeyer type operators

Antonio Jesús López Moreno

Abstract

We present several results for a class of Durrmeyer type operators that generalizes some other sequences of operators that have appeared recently in the literature. In particular we study asymptotic expressions and localization results.

Keywords: Durrmeyer operators, asymptotic formula.
AMS Classification: 41A36.

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Asymptotics of the Christoffel functions on the unit ball in the presence of a mass on the sphere*

Clotilde Martínez and Miguel A. Piñar

Abstract

We study a family of mutually orthogonal polynomials on the unit ball with respect to an inner product which includes a mass uniformly distributed on the sphere. First,using the representation formula for these polynomials in terms of spherical harmonics analytic properties will be deduced. Finally, we analyse the asymptotic behaviour of the Christoffel functions.

Keywords: orthogonal polynomials, Uvarov modification, Christoffel functions.
AMS Classification: 42C05, 33C50

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Uniform convergence of Hermite–Fejér interpolation at Laguerre zeros

Giuseppe Mastroianni

Abstract

This topic has received few attention in the literature. G. Szegő obtained a first partial result in [2]. In [3] P. Vértesi gave a quantitative estimate of the error under the assumptions posed by Szegő. In [1] J. Szabados studied the convergence–divergence of these polynomials under less restrictive hypotheses.

In this talk we are going to show that a slight modification of the Hermite–Fejér operator leads to more precise results on convergence and error estimate.

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Iterated Kantorovich method for integral equation of the second kind on the real line*

Abdelaziz Mennouni

Abstract

Iterated Kantorovich method is formulated and justified for the approximate solution of integral equation of the second kind on the real line of the form

$$x(s) - \int_{-\infty}^{\infty} k(s,t)x(t)\omega(t)dt = f(s), \quad s \in \mathbb{R},$$

where $\omega(t) := e^{-t^2}$ is the weight function and $k(\cdot,\cdot)$ is a Fredholm kernel, our approach is based on a sequence of orthogonal finite rank projections.

The convergence analysis is discussed and associated theorems are considered in this work.

Some numerical examples are presented to illustrate the theoretical results where we show the effectiveness of the method.

Keywords: approximation, integral equation, iterated Kantorovich method, finite rank projections, Hermite polynomials.

AMS Classification: 41A30, 45E05, 45J05.

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Some summability methods and Korovkin type approximation theorems

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Abstract

Korovkin type approximation theorems are useful tools to check whether a given sequence $(L_n)_{n\geq 1}$ of positive linear operators on $C[0,1]$ of all continuous functions on the real interval $[0,1]$ is an approximation process. That is, these theorems exhibit a variety of test functions which assure that the approximation property holds on the whole space if it holds for them. Such a property was discovered by Korovkin in 1953 for the functions 1, x and x^2 in the space $C[0,1]$ as well as for the functions 1, cos and sin in the space of all continuous 2π-periodic functions on the real line. In this talk, we use the notion of almost convergence and statistical convergence to prove the Korovkin type approximation theorems for the test functions 1, e^x, e^{2x}.

Keywords: Korovkin theorem, almost convergence, statistical convergence.
AMS Classification: 40A35, 41A36.

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Orthogonal polynomials and Lagrange interpolation for exponential weights on the real semiaxis*

Incoronata Notarangelo

Abstract

The weighted polynomial approximation for functions defined on $(0, +\infty)$ which can grow exponentially at 0^+ and/or $+\infty$ has been considered in the literature only recently [2, 3, 4, 5, 6]. In particular, estimates for the best weighted approximation have been proved.

Here, in order to construct Lagrange approximation processes, we consider the orthonormal system $\{p_m(w)\}_m$ associated to weight

$$w(x) = x^\gamma \mathrm{e}^{-x^{-\alpha}-x^\beta} \qquad x \in (0, +\infty)\,,$$

where $\alpha > 0$, $\beta > 1$ and $\gamma \geq 0$. We observe that the weight w can be seen as a combination of a Pollaczeck-type weight $\mathrm{e}^{-x^{-\alpha}}$ and a Laguerre-type weight $x^\gamma \mathrm{e}^{-x^\beta}$. Nevertheless the properties of $\{p_m(w)\}_m$ cannot be deduced from previous results concerning these two weights.

We show that the weight w can be reduced to a weight belonging to the Levin–Lubinsky class $\mathcal{F}(C^2+)$ [1] and then we obtain estimates for the polynomials $p_m(w)$, their zeros and the associated Christoffel function.

Finally, we apply these results to the study of the convergence of Lagrange interpolation processes based on the zeros of $p_m(w)$.

Keywords: orthogonal polynomials, weighted polynomial approximation, Pollaczeck–Laguerre weights, nonstandard exponential weights, unbounded interval, real semiaxis.
AMS Classification: 33C47, 33C52, 41A05.

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Best polynomial approximation on the unit ball*

Miguel Piñar and Yuan Xu

Abstract

The purpose of this talk is to show some basic properties of the best approximation by polynomials of degree at most n on the unit ball $\mathbb{B}^d$ in $\mathbb{R}^d$. For the standard Gegenbauer weight function

$$\varpi_\mu(x) = (1 - \|x\|^2)^\mu, \qquad \mu > -1, \quad x \in \mathbb{B}^d,$$

let $\|\cdot\|_\mu$ denote the norm in $L^2(\varpi_\mu; \mathbb{B}^d)$, then we determine the connection between the error of best approximation of a function in the Sobolev space

$$W_2^s(\varpi_\mu, \mathbb{B}^d) := \{f \in L^2(\varpi_\mu, \mathbb{B}^d) : \partial^{\mathbf{m}} f \in L^2(\varpi_{\mu+|\mathbf{m}|}, \mathbb{B}^d),\ |\mathbf{m}| \le s,\ \mathbf{m} \in \mathbb{N}_0^d\}.$$

and the error of best approximation of the corresponding derivatives. The case $d = 1$ is classical, the extension of this result to higher dimensions, even in the ball case, contains some subtle difficulties. In fact, to obtain our estimates we need the concourse of standard and angular derivatives.

Let $E_n(f)_\mu$ be the error of best approximation by polynomials of degree at most n in the space $L^2(\varpi_\mu, \mathbb{B}^d)$. Our main result shows that, for $s \in \mathbb{N}$,

$$E_n(f)_\mu \le \frac{c}{n^{2s}}[E_{n-2s}(\Delta^s f)_{\mu+2s} + E_n(\Delta_0^s f)_\mu],$$

where Δ and Δ_0 are the Laplace and Laplace-Beltrami operators, respectively. We also derive a bound when the right hand side contains odd order derivatives.

The proof of these results are based on the Fourier expansions in orthogonal polynomials with respect to the Gegenbauer weight functions on the unit ball. The key ingredients are the commuting relations between partial derivatives and the orthogonal projection operators, and explicit formulas for an explicit basis of orthogonal polynomials and their derivatives. The relations between the orthogonal polynomials and their derivatives depend on corresponding relations for an explicit basis of spherical harmonics, which are of independent interest.

Keywords: best approximation, polynomials, orthogonal polynomials, unit ball.
AMS Classification: 33C50, 42C10.

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The first two Zolotarev cases in the Erdös-Szegö solution to a Markov-type extremal problem of Schur

Heinz-Joachim Rack

Abstract

hur's [12] Markov-type extremal problem of 1919 asks to determine

$$\sup_{|\xi|\leq 1} \sup_{P_n \in \boldsymbol{B}_{n,\xi,2}} |P_n^{(1)}(\xi)| \text{ , where } \boldsymbol{B}_{n,\xi,2} := \{P_n \in \boldsymbol{B}_n : P_n^{(2)}(\xi) = 0\} \subset \boldsymbol{B}_n := \{P_n : |P_n(x)| \leq 1 \text{ for } |x| \leq 1\} \text{ and } P_n \text{ is an algebraic polynomial of degree } \leq n. \quad (i)$$

Erdös and Szegö [3] found in 1942 that (i) will be attained, for $n \geq 4$, if $\xi = \pm 1$ and $P_n \in \boldsymbol{B}_{n,\xi,2}$ is a (unspecified) member of the 1-parameter family of hard-core Zolotarev polynomials $Z_{n,t} \in \boldsymbol{B}_n$ (for $n = 3$ the Chebyshev polynomial T_3 is extremal at $\xi = 0$), see also [4]. For the first Zolotarev case, $n = 4$, we obtain, based on preliminary work in [3] and cross-checked with results from [8, p. 70] and [9, p. 357], an explicit and more detailed solution to Schur's problem, see [10]: We get the explicit analytical formula for the constant (i) and for the optimal parameter $t = t*$, and even get the explicit algebraic power form representation of the extremizer $Z_{4,t*}$ in (i). It turns out that, for $n = 4$, the numerical value for the constant (i), as given in [3, Formula (7.9)], is biased. To amend on the second Zolotarev case, $n = 5$, we rely on the quite recently discovered algebraic power form representation of $Z_{5,t}$ in [5, 6] and find that a detailed solution to Schur's problem intrinsically cannot be as smooth as for $n = 4$ (and the cases $n > 5$ still remain arcane), see [11]: For it turns out that the optimal parameter $t = t*$, which singles out among all $Z_{5,t}$ the extremizer $Z_{5,t*}$ in (i), is a dedicated zero of some minimal P_{10} whose Galois group is not solvable so that $t*$ cannot be expressed by radicals. Hence we resort to an arbitrarily precise numerical solution. Furthermore, we contribute to the fuller picture of $Z_{5,t}$ by providing its critical points and the concrete implementation of the Abel-Pell equation. Then we turn to a generalization of (i) to higher derivatives, as recently proposed by Shadrin [13], and obtain solutions, for $n \in \{4, 5\}$, analogous to those for the first derivative. Finally we describe, for $n = 5$, three new algebraic approaches to Zolotarev's so-called first problem [1] which Zolotarev himself [14] had originally solved (for arbitrary n) by means of elliptic functions. We exemplify one of these approaches by determining the rather involved explicit coefficients of the optimal quintic polynomial (in power form) which, among all P_5 with the first two leading coefficients set equal to 1, deviates least (in the uniform norm) from the zero function on the interval $[-1, 1]$. This result improves on [2, p. 186] and [7, p. 937].

Keywords: Abel-Pell equation, critical points, Erdös, extremal problem, Grasegger, inequality, Markov, polynomial, quartic, quintic, Schur, Shadrin, Szegö, Vo, Zolotarev.

AMS Classification: 26C05, 26D10, 41A10.

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Szasz-Durrmeyer operators involving Boas-Buck polynomials of blending type

Manjari Sidharth and P.N. Agrawal

Abstract

Szász [2] generalized the Bernstein polynomials to the infinite interval as

$$S_n(f;x) = e^{-nx}\sum_{k=0}^{\infty}\frac{(nx)^k}{k!}f\left(\frac{k}{n}\right), \quad \forall x\in[0,\infty) \text{ and } n\in\mathbb{N}.$$

In [1], Sucu et al. introduced the Szasz operators involving Boas-Buck type polynomials as follows:

$$B_n(f;x) := \frac{1}{A(1)G(nxH(1))}\sum_{k=0}^{\infty}p_k(nx)f\left(\frac{k}{n}\right), \quad x\geq 0,\ n\in\mathbb{N}, \tag{1}$$

where generating function of the Boas-Buck type polynomials is given by

$$A(t)G(xH(t)) = \sum_{k=0}^{\infty}p_k(x)t^k,$$

and $A(t), G(t)$ and $H(t)$ are analytic functions

$$\begin{aligned} A(t) &= \sum_{k=0}^{\infty}a_k t^k, \ (a_0\neq 0), \\ G(t) &= \sum_{k=0}^{\infty}g_k t^k, \ (g_k\neq 0), \\ H(t) &= \sum_{k=1}^{\infty}h_k t^k, \ (h_1\neq 0). \end{aligned}$$

Motivated by the above work, in the present paper we define the Durrmeyer type operators based Boas-Buck type polynomials as follows:
For $\gamma > 0$, let $C_\gamma[0,\infty) := \{f \in C[0,\infty) : |f(t)| \leq M(1+t^\gamma),\ \text{for some}\ M > 0\}$. Then for a function $f \in C_\gamma[0,\infty)$, we define

$$M_n(f;x) := \frac{1}{A(1)G(nxH(1))}\sum_{k=1}^{\infty}\frac{p_k(nx)}{B(k,n+1)}\int_0^\infty \frac{t^{k-1}}{(1+t)^{n+k+1}}f(t)dt + \frac{a_0b_0}{A(1)G(nxH(1))}f(0),$$

where $B(k,n+1)$ is the beta function and $x \geq 0,\ n \in \mathbb{N}$.

Alternatively, we may write the above operatoras

$$M_n(f;x) := \int_0^\infty W(n,x,t)f(t)dt, \tag{2}$$

where

$$W(n,x,t) := \frac{1}{A(1)G(nxH(1))}\sum_{k=1}^{\infty}\frac{p_k(nx)}{B(k,n+1)}\frac{t^{k-1}}{(1+t)^{n+k+1}} + \frac{a_0b_0}{A(1)G(nxH(1))}\delta(t),$$

and $\delta(t)$ being the Dirac-delta function.

The present paper deals with the Szasz operators involving Boas-Buck type polynomials which include Brenke-type polynomials, Sheffer polynomials and Appell polynomials. We establish the moments of the considered operators and a Voronvskaja type asymptotic theorem and then proceed to study the convergence of the operators with the help of Lipschitz type space and weighted modulus of continuity. Furthermore, we obtain a direct approximation theorem with the aid of unified Ditzian-Totik modulus of smoothness.

Keywords: Lipschitz type space, Ditzian-Totik modulus of smoothness, weighted modulus of continuity.
AMS Classification: 41A10, 41A25, 41A36.

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Polynomial inequalities with nonsymmetric weights*

András Kroó and József Szabados

Abstract

Remez-, Schur-, and Bernstein-type weighted polynomial inequalities are discussed with nonsymmetric weights like

$$w(x) = \begin{cases} |x|^\alpha, & \text{if} \quad -1 \le x \le 0, \\ x^\beta, & \text{if} \quad 0 < x \le 1 \end{cases} \qquad (\beta \ge \alpha \ge 0)\,.$$

Keywords: nonsymmetric weights, polynomial inequalities.
AMS Classification: 41A17.

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Rectangular summability and Lebesgue points of higher dimensional Fourier transforms*

Ferenc Weisz

Abstract

Three types of rectangular summability of higher dimensional Fourier transforms are investigated with the help of an integrable function θ, the unrestricted summability, the summability over a cone and over a cone-like set. We introduce the concept of different Lebesgue points and show that almost every point is a Lebesgue point of f from the Wiener amalgam space $W(L_1, \ell_\infty)(\mathbb{R}^d)$. We give three generalizations of the well known Lebesgue's theorem for the summability of higher dimensional Fourier transforms. More exactly, under some conditions on θ we show that the different types of summability means of a function $f \in W(L_1, \ell_\infty)(\mathbb{R}^d) \supset L_1(\mathbb{R}^d)$ converge to f at each Lebesgue point.

Keywords: Fourier transforms, Fejér summability, θ-summability, Marcinkiewicz summability, Lebesgue points, strong summability.

AMS Classification: Primary 42B08; Secondary 42A38, 42A24, 42B25.

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