Polynomial operators and local smoothness classes on the unit interval, II†

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Abstract

We prove the existence of quadrature formulas exact for integrating high degree polynomials with respect to Jacobi weights based on scattered data on the unit interval. We also obtain a characterization of local Besov spaces using the coefficients of a tight frame expansion.

Keywords: Local approximation, localized Jacobi kernels, quadrature formulas, tight frames, local Besov spaces.

MSC: Primary 41A55; Secondary 41A25, 42B35.

§1. Introduction

It is well known that a major drawback of polynomial approximation is that polynomials cannot be localized; a polynomial of degree $n$ is completely determined by its values at $n + 1$ points on an interval, howsoever small. Another example is the following. If $P_n^*$ is a best polynomial approximation of degree at most $n$ to the function $f(x) = |x|$ on $[-1, 1]$, then there are at least $n + 2$ points $y_j$ on $[-1, 1]$, where $|f(y_j) - P_n^*(y_j)| \geq cn^{-1}$, where $c$ is a positive constant independent of $n$ and the points $y_j$. Moreover, as $n \to \infty$, these points become dense on $[-1, 1]$. Thus, even though the function is piecewise analytic, the

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