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New results and open problems in approximation of convex and star-like surfaces by algebraic level surfaces[†]

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Abstract

In this note we consider the problem of approximation of convex surfaces in $\mathbb{R}^d$ by level surfaces of algebraic polynomials. P.C. Hammer [3] verified that any convex surface in $\mathbb{R}^d$ can be approximated by a convex level surface of an algebraic polynomial. We shall extend Hammer's approximation theorem by showing that the order of approximation of convex surfaces by convex algebraic surfaces of degree n is bounded from above by $\frac{\log n}{n}$, and, in addition, this bound is essentially independent of the convex surfaces approximated. Moreover, it will also be shown that the rate of approximation of $\mathbf{0}$-symmetric convex polytopes by homogeneous algebraic surfaces of degree n can not be better, than $\frac{1}{n \log n}$. Furthermore, it will be verified that whenever the boundary of a $\mathbf{0}$-symmetric convex body is C^α-smooth with $1 < \alpha \leq 2$ then the order of approximation of this surface by homogeneous algebraic level surfaces of degree n is bounded from above by $O(\frac{\log^\alpha n}{n^\alpha})$ and this order is sharp up to the log factor.

Keywords: Level surfaces of algebraic polynomials and homogeneous polynomials, Hausdorff distance, convex bodies, maximal ellipsoids, star-like domains.

MSC: Primary 41A17; Secondary 52A27.

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