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on Approximation

Least squares approximation and extremal properties

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Abstract

This paper deals with best approximation in semi-inner product spaces H where functions of H are approximated in the underlying semi-norm by functions of a finite dimensional subspace U . In the approximation problems there are also used regularization terms, e.g. the well-known Tikhonov regularization is considered. The approximation problems are transformed to quadratic optimization problems. Characterization and uniqueness results for the solutions are given. The solutions of these best approximations satisfy important extremal properties. We give several results of this type. We apply our results to more special problems, e.g. L_2 -approximation from finite dimensional subspaces U and subspaces U which are spanned by translates of a conditionally reproducing kernel.

Keywords: Least squares approximation, regularization, conditionally positive definite functions.

MSC: Primary 41A15; Secondary 65D10, 42A82.

§1. Introduction

In this paper we first study quadratic optimization problems of the form:

Minimize

$$d(a) = \frac{1}{2}a^T Aa + p^T a + \beta$$

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