



Moduli of smoothness of splines and applications in constrained approximation[†]

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Abstract

In this paper, we generalize [9, Theorem 4.1] to be applicable to the classes of (almost) weakly/nearly (co)- k -monotone functions and discuss some applications and open problems.

Keywords: constrained approximation, moduli of smoothness, orders of approximation, (almost) weakly/nearly (co)- k -monotone functions..

MSC: Primary 41A29; Secondary 41A10, 41A25.

§1. Introduction and main results

Let $\mathcal{S}_r(\mathbf{z}_n)$ be the space of all piecewise polynomial functions (ppf's) of degree r (order $r + 1$) with the knots $\mathbf{z}_n := (z_i)_0^n$, $-1 =: z_0 < z_1 < \cdots < z_{n-1} < z_n := 1$. In other words, we say that $s \in \mathcal{S}_r(\mathbf{z}_n)$ if, on each interval (z_i, z_{i+1}) , $0 \leq i \leq n - 1$, s is in Π_r , where Π_r denotes the space of algebraic polynomials of degree $\leq r$.

For a partition $\mathbf{z}_n := \{z_0, \dots, z_n \mid -1 =: z_0 < z_1 < \cdots < z_n := 1\}$, let $J_j := [z_j, z_{j+1}]$ with $z_j := -1$, $j < 0$, and $z_j := 1$, $j > n$, and $|J| := \text{meas } J$.

Given an absolute constant Δ we say that \mathbf{z}_n is “ Δ -quasi-uniform” if

$$\Delta(\mathbf{z}_n) := \max_{0 \leq j \leq n-1} |J_j| / \min_{0 \leq j \leq n-1} |J_j| \leq \Delta,$$

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