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On the number of maximal planes of GC_2 sets in $\mathbb{R}^3$

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Abstract

Sets of interpolation nodes satisfying the geometric condition (GC) of Chung and Yao are considered. A maximal hyperplane for a GC_n set (a GC set of degree n) in $\mathbb{R}^d$ contains $\binom{n+d-1}{d-1}$ points of that set. There exist several conjectures about the number of maximal hyperplanes for a GC set. In the bivariate case, Gasca and Maeztu in 1982 conjectured that for every GC_n set there exists at least a maximal line. This has been proved for $n \leq 4$. Later on Carnicer and Gasca proved that for $n \leq 4$, and for every GC_n set, there exist at least 3 maximal lines and conjectured that this holds for $n > 4$. De Boor extended these conjectures to $\mathbb{R}^d$: at least a maximal hyperplane as extension of the first one and at least $d + 1$ maximal hyperplanes as extension of the second one. The same author proved that the second conjecture does not hold for $d > 2$, showing a counterexample with $d = 3$, $n = 2$ and only 3 maximal hyperplanes. Recently, Apozyan et al. proved that for $d = 3$, $n = 2$, there exists at least one maximal hyperplane. In the present paper it is proved that, in fact, when $d = 3$, $n = 2$, there exist at least 3 maximal hyperplanes.

Keywords: multivariate interpolation, poised set, geometric characterization, Gasca-Maeztu conjecture, maximal hyperplane.

MSC: Primary 41A05, 41A63; Secondary 41A10, 65D05.