Computation of Gauss-type quadrature formulas with some preassigned nodes†

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Abstract

When dealing with the approximate calculation of weighted integrals over a finite interval \([a, b]\), Gauss-type quadrature rules with one or two prescribed nodes at the end points \(\{a, b\}\) are well known and commonly referred as Gauss-Radau and Gauss-Lobatto formulas respectively. In this regard, efficient algorithms involving the solution of an eigenvalue problem for certain tri-diagonal (Jacobi) matrices are available for their computation. In this work a further step will be given by adding to the above quadratures an extra fixed node in \((a, b)\) and providing similar efficient algorithms for their computation. This will be done by passing to the unit circle and taking advantage of the so-called Szegő-Lobatto quadrature rules recently introduced in [27] and [6].

Keywords: Szegő-type quadrature formulas, Gauss-type quadrature formulas, para-orthogonal polynomials, Jacobi matrices.

MSC: Primary 41A55, 42C05; Secondary 65D30, 65F15.

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