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On Bernstein and Chebyshev type problems for k-monotone polynomials†

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Abstract

In a recent paper [4], we initiated the study of Markov type inequalities for the so called k-monotone polynomials of degree at most n, whose first k derivatives are nonnegative in the interval considered. The exact constant of Markov type inequality was found in case of first derivative when the underlying norm is uniform or L_1. Moreover it was shown that in general for derivatives of order j the sharp order of magnitude of Markov constants is $\left(\frac{n^2}{k}\right)^j$.

In the present paper we continue the study of this subject by establishing asymptotically sharp Bernstein type inequalities for derivatives of k-monotone polynomials. We shall show that the order of magnitude of Bernstein factors is $\left(\varphi\left(\frac{k}{n^2}\right)\frac{n^2}{k}\right)^j$, with the size of the derivatives on $[-1,1]$ being measured in weighted norms with weight $\varphi(1-x)$, where φ is such that $\varphi(x)$ is increasing and $\varphi(x)/x$ is decreasing.

In addition we also give an exact solution to the Chebyshev type problem of finding monic k-monotone polynomials of minimal L_1 norm. It turns out that just as in the case of Markov type inequalities for k-monotone polynomials, these extremal polynomials are related to certain Jacobi polynomials. The result for L_∞ norm of Bernstein will also be recovered.

Keywords: Bernstein inequality, Chebyshev problem, k-convex polynomial.

MSC: 41A17.

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