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Asymptotic formulae for positive linear operators: direct and converse results

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Abstract

This paper is concerned with asymptotic formulae for sequences of positive linear operators acting on weighted spaces of continuous functions defined on a real interval. The main result provides a characterization of those continuous functions for which a pointwise asymptotic formula holds true. The method is essentially based on a characterization of the domains of generators of C_0 -semigroups in terms of asymptotic formulae. Finally, several applications concerning, among others, Kantorovich operators, exponential operators, Gauss-Weierstrass operators and a sequence of operators related to the Black-Scholes equation are discussed as well.

Keywords: Asymptotic formula, positive approximation process, local saturation, C_0 -semigroup of operators.

MSC: Primary 41A36, 41A40, 41A80,47D07; Secondary 35K15.

§1. Introduction

The study of asymptotic formulae for positive linear operators is a topic of current interest in approximation theory. Such formulae play an important role in the analysis of the saturation properties of approximation processes and in the approximation of strongly continuous semigroups by means of iterates of operators. To this regard,

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