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Old and New Geronimus Type Identities for Real Orthogonal Polynomials[†]

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Abstract

Let μ be a positive measure on the real line, with orthogonal polynomials $\{p_n\}$ and leading coefficients $\{\gamma_n\}$. The Geronimus type identity

$$\frac{1}{\pi}\left|\mathrm{Im} z\right| \int_{-\infty}^{\infty} \frac{P(t)}{\left|z p_n(t) - p_{n-1}(t)\right|^2} dt = \frac{\gamma_{n-1}}{\gamma_n} \int P(t)\ d\mu(t),$$

valid for all polynomials P of degree $\leq 2n-2$ has known analogues within the theory of orthogonal rational functions, though apparently unknown in the theory of orthogonal polynomials. We present new proofs of this and its generalization,

$$\int_{-\infty}^{\infty} \frac{P(t)}{p_n^2(t)} h\left(\frac{p_{n-1}}{p_n}(t)\right) dt = \frac{\gamma_{n-1}}{\gamma_n} \left(\int_{-\infty}^{\infty} h(t)\, dt\right) \left(\int P(t)\ d\mu(t)\right),$$

valid for any $h \in L_1(\mathbb{R})$.

Keywords: Orthogonal Polynomials on the real line, Geronimus type formula, Poisson integrals.

MSC: Primary 42C05; Secondary 41A99.

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