Some inequalities related to the moduli of smoothness of polynomials

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Abstract

We present some inequalities related to the moduli of smoothness of trigonometric and algebraic polynomials. In the case of algebraic polynomials we consider the Ditzian-Totik type moduli of smoothness. Some inequalities are similar to the ones due to Nikolskii and Stechkin and they are also a connected with Bernstein type inequalities.

Keywords: Polynomial inequalities, modulus of smoothness, Bernstein’s inequality, Nikolskii-Stechkin’s inequality.

MSC: Primary 41A17; Secondary 26D05.

§1. Introduction

Let $C[-1,1]$ be the space of all real continuous functions $f$ endowed with the norm $\|f\| = \sup_{x \in [-1,1]} |f(x)|$. For $1 \leq p < \infty$, let $L_p[-1,1]$ be the usual space of Lebesgue integrable functions with the norm $\|f\|_p = \left(\int_{-1}^{1} |f(x)|^p \, dx \right)^{1/p}$. To simplify notations we sometimes write $L_\infty[-1,1]$ instead of $C[-1,1]$. The family of all algebraic polynomials of degree not greater than $n$ is denoted by $\Pi_n$.

The function $\varphi(x) = \sqrt{1-x^2}$, $x \in [-1,1]$, will be used throughout the paper. For $r \in \mathbb{N}$ and $f \in L_p[-1,1]$ ($1 \leq p \leq \infty$) the Ditzian-Totik modulus of smoothness of order $r$