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A duality formula for Chebyshevian divided differences and blossoms

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Abstract

Extending a simple result of the polynomial framework, we establish an interesting formula connecting divided differences in Extended Chebyshev spaces and blossoms in the dual spaces.

Keywords: Extended Chebyshev spaces, weight functions, generalised derivatives, Chebyshevian divided differences, blossoms.

MSC: 41A05, 41A50, 65D05, 65D17.

§1. Introduction

Extended Chebyshev spaces being the natural generalisations of polynomial spaces, it is quite logical to try and find the analogue of every single interesting result known in the polynomial framework. Such attempts are all the more worthwhile as quite often the Chebyshevian approach offers new insights into the polynomial case.

This is precisely the purpose of the present paper, starting from $(n-1)$th order divided differences of the monomial $G_n(x) := x^n$. Given any real numbers $x_1, \dots, x_n$, and any sufficiently differentiable function F defined on an interval containing the x_i's, let $[x_1, \dots, x_n]F$ denote the $(n-1)$th order divided difference of F based on $(x_1, \dots, x_n)$. Using the fact that $G_{n+1} = G_n G_1$ along with the Leibniz' formula for divided differences, it is easy to prove by induction that

$$[x_1, \dots, x_n]G_n = x_1 + \dots + x_n, \quad x_1, \dots, x_n \in \mathbb{R}. \tag{1.1}$$