



ISSN: 1889-3066

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Web site: jja.ujaen.es

Jaen J. Approx. 3(2) (2011), 173–191

Jaen Journal

on Approximation

Refinable ripples with dilation 3

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Abstract

We present a class of one parameter refinable functions with dilation 3 having the same smoothness as the B-splines. After constructing these functions we analyze their main properties. Some examples are also displayed.

Keywords: refinable function, B-spline, Fourier transform, optimal basis.

MSC: Primary 41A15, 65T60; Secondary 41A25, 65D07.

Communicated by

K. Jetter

Received

May 22, 2011

Accepted

November 4, 2011

§1. Introduction

A main problem in approximation theory is the construction of approximations that are close to the graph of the function to be approximated and preserve its shape. Shape preservation is as well crucial in the design of curves or surfaces as required in CAGD applications. In this context, a prominent role is played by the *total positivity* of the functional basis, by which the approximation is generated. If the totally positive basis is formed by the integer translates of a given function, i.e. the totally positive system is given by $\{\varphi(x - k)\}_{k \in \mathbb{Z}}$, the function φ is said to be a *ripple* (see [8]).

A large class of refinable ripples with dilation $M = 2$, here denoted by GP ripples, has been introduced in [10]. The GP ripples depend on one or more parameters that affect their shapes. For instance, particular values of the parameters give rise to the B-splines on integer knots, otherwise the GP ripples are not piecewise polynomial functions. Nevertheless, GP ripples retain the salient properties of the B-splines, namely compact support, symmetry, shape preserving properties. For this reason, they can be considered