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On Padé approximants and the ε–algorithms for vectors in 2 and 3 dimensions

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Abstract

After reminding what Padé approximants are and how they can be recursively computed by the ε-algorithm, relations between the results produced by the scalar ε-algorithm applied to a sequence of complex numbers and by the vector ε-algorithm applied to the two-dimensional vectors formed by their real and imaginary parts are given. This result is then extended to sequences of quaternions and three-dimensional vectors. Negative results and an open question end the paper.

Keywords: extrapolation, convergence acceleration, Padé approximation.

MSC: Primary 65B05, 41A21; Secondary 65B10, 65D05.

Padé approximants are rational functions whose series expansions in ascending powers of the variable agree with a given formal power series as far as possible. They have been extensively studied and used for the solution of problems in numerical analysis, applied mathematics, physics, chemistry, etc., and a huge literature on the topic exists; see, for example, [1, 8].

On the other end, let us consider a sequence which converges to its limit. In many applications, one has no access to the way in which the sequence is built, it is a black box. If the convergence is slow, it is possible to transform the sequence into a new sequence which, under some assumptions, converges to the same limit faster than the initial sequence [6]. A well-known and quite efficient such sequence transformation was introduced by Shanks [15], and it can be implemented by the ε-algorithm of Wynn [16]. If the sequence to be