A contribution to the
Gr¨unwald–Marcinkiewicz theorem†

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Abstract
This paper is a certain generalization of the Gr¨unwald–Marcinkiewicz theorem revealing its connection to a process defined by S. N. Bernstein.

Keywords: interpolation, operator norm, divergence, Grünwald-Marcinkiewicz theorem, Bernstein operator.

MSC: 41A05.

§1. Introduction

1.1. We begin with some definitions and notations. Ĉ stands for the space of $2\pi$-periodic continuous functions, $T_m$ denotes the space of trigonometric polynomials of degree at most $m$ of the form $\frac{a_0}{2} + \sum_{k=1}^{m}(a_k \cos k\vartheta + b_k \sin k\vartheta)$, $a_k, b_k$ being reals. If $\Theta = \{\vartheta_{km}, k = 0, \ldots, 2m, m = 1, 2, \ldots\} \subset [0, 2\pi)$ is an interpolatory matrix with

$$0 \leq \vartheta_{0m} < \vartheta_{1m} < \cdots < \vartheta_{2m,m} < 2\pi,$$

then the uniquely defined $m^{th}$ trigonometric interpolatory polynomial for $f \in \hat{C}$ is

$$T_m(f, \Theta, \vartheta) = \sum_{k=0}^{2m} f(\vartheta_{km}) t_{km}(\Theta, \vartheta),$$

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