



# On normality of orthogonal polynomials<sup>†</sup>

Vilmos Totik

## Abstract

We extend some recent results of Martínez-Finkelshtein and Simon about measures  $\mu$  on the unit circle for which the corresponding orthonormal polynomials  $\varphi_n$  have the so called normal behavior:  $\|\varphi'_n\|/n \rightarrow 1$ .

**Keywords:** orthogonal polynomials on the unit circle, doubling weights, normal behavior.

**MSC:** 42C05.

Let  $\mu$  be a Borel-measure on the unit circle  $\mathbb{T}$  (with support that contains infinitely many points) and let  $\varphi_n(z) = \kappa_n z^n + \dots$  be the orthonormal polynomials associated with  $\mu$ . Thus,

$$\int \varphi_n \overline{\varphi_m} d\mu = 0 \quad \text{if } n \neq m, \quad \text{and} \quad \int |\varphi_n|^2 d\mu = 1.$$

It is a simple fact due to the orthogonality, that here

$$\frac{1}{\kappa_n^2} = \inf \left\{ \int |P_n|^2 d\mu \mid P_n(z) = z^n + \dots \right\}, \quad (1)$$

<sup>†</sup>Supported by the TAMOP-4.2.1/B-09/1/KONV-2010-0005 project.

Communicated by  
G. López Lagomasino

Received  
October 30, 2011  
Accepted  
January 31, 2012