ISSN: 1889-3066
© 2012 Universidad de Jaén
Web site: jja.ujaen.es

Jaen J. Approx. 4(2) (2012), 121–136

Jaen Journal on Approximation

Characterization of n-independent sets with no more than $3n$ points[†]

H. Hakopian and A. Malinyan

Communicated by
K. Jetter

Received
June 21, 2012
Accepted
October 2, 2012

Abstract

Let Π_n denote the space of all bivariate polynomials of total degree not exceeding n. We study the n-independence of point sets. These sets $\mathcal{X}$ are characterized by the fact that the dimension of the space $\mathcal{P}_{\mathcal{X}} = \{p \in \Pi_n : p(\mathbf{x}) = 0, \forall \mathbf{x} \in \mathcal{X}\}$ equals $\dim \Pi_n - \#\mathcal{X}$. Besides, polynomial interpolation of degree n is solvable only with these sets. Moreover, the n-independent sets are exactly the subsets of Π_n-poised sets. In this paper we characterize all n-independent sets with no more than $3n$ points.

Keywords: Fundamental polynomial, independent point set.

MSC: Primary 41A05; Secondary 41A63, 14H50.

§1. Introduction, independent point sets

Let us denote by $\Pi_n = \Pi_n(\mathbb{R}^2)$ the space of all bivariate algebraic polynomials of total degree not exceeding n:

$$\Pi_n = \left\{ p(x,y) = \sum_{i+j \le n} a_{ij} x^i y^j,\ a_{ij} \in \mathbb{K} \right\},$$

[†]The first author is supported by State Committee of Science of Armenia, grant # 11-1A290.