Cheney and Sharma type operators and Korovkin test functions

Jorge Bustamante and Miguel A. Jiménez Pozo

Abstract

Hereby we define a class of sequences of positive linear operators that contains the ones introduced by Cheney and Sharma in 1964, as well as several others based upon the same idea of the classical Meller-König and Zeller operators. The sequences included in this class share the property of being associated to functional differential equations which let us the possibility of explicitly evaluating the Korovkin test functions as solutions of the equations. After a unified study of different approximation problems for sequences within this class, we apply the constructed theory to explicit sequences of operators from which we obtain old and new results as well.

Keywords: Cheney and Sharma operators, positive linear operators, Korovkin test functions, rate of convergence.

MSC: 41A17, 41A25.

§1. Introduction

In [6] Cheney and Sharma introduced a slight modification of the Meyer-König and Zeller operators originally given in [12] by defining for every $f \in C[0, 1]$,

$$M_n(f, x) = (1 - x)^{n+1} \sum_{k=0}^{\infty} f\left(\frac{k}{n+k}\right) \binom{n+k}{k} x^k,$$  \hspace{1cm} (1.1)