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On Trigonometric Sums in Two Variables

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Abstract

We study inequalities for two closely related trigonometric sums in two variables and prove the following theorems:

(i) For all $n \geq 1$ and for all x, y with $0 < x + y < 2\pi$, $0 < x - y < 2\pi$ we have

$$\sum_{k=1}^{n} \frac{\sin(kx)\cos(ky)}{k} \leq \alpha \cdot \Big(\frac{1}{\sin\frac{x+y}{2}} + \frac{1}{\sin\frac{x-y}{2}} \Big)$$

with the best possible constant factor $\alpha = 2\sqrt{3}/9$.

(ii) The inequality

$$0 \leq \beta \sin x \cos y + \sum_{k=2}^{n} \frac{\sin(kx)\cos(ky)}{k+1}$$

holds for all $n \geq 1$ and for all x, y with $0 < x + y < \pi$, $0 < x - y < \pi$ if and only if $\beta \geq 3/2 - \log 2$.

Keywords: trigonometric sums, Fejér-Jackson inequality, Vietoris' theorem, best possible constant, inequalities.

MSC: 26D05, 42A05.

§1. Introduction

The inequality

$$0 < \sum_{k=1}^{n} \frac{\sin(kx)}{k} \quad (n = 1, 2, ...; 0 < x < \pi) \tag{1.1}$$