Maximum entropy of nonnegative functions subject to convex constraints†

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Abstract

In this paper we discuss the problem of maximizing the entropy of a nonnegative function on a $\sigma$-finite measure space subject to convex constraints. For a finite number of moment conditions, the search for an essential set reduces the problem to a system of nonlinear equations. We also pose a finite dimensional dual optimization problem whose solution is related to the solution of the primal problem. Finally we provide an abstract unifying approach in order to include maximum entropy problems when a finite number of marginals of a probability density are given. Some illustrative examples are provided.

Keywords: entropy, moment conditions, marginals.
MSC: 41A29, 94A17.

§1. Introduction

The mathematical notion of entropy is ubiquitous in sciences and engineering. For example, it appears in statistical mechanics, physical chemistry, multivariate statistics, electrical engineering, language modeling in speech recognition and information theory [4].

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