Determining singularities using rows of Padé-orthogonal approximants

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Abstract

Starting from the orthogonal polynomial expansion of a function $F$ corresponding to a finite positive Borel measure with infinite compact support, we study the asymptotic behavior of certain associated rational functions (Padé-orthogonal approximants). We obtain both direct and inverse results relating the convergence of the poles of the approximants and the singularities of $F$. Thereby, we obtain analogues of theorems by E. Fabry, R. de Montessus de Ballore, V. I. Buslaev, and S. P. Suetin.

Keywords: Padé approximants, orthogonal polynomials, Fabry’s theorem, Montessus de Ballore’s theorem.

MSC: Primary 30E10, 41A27; Secondary 41A21.

§1. Introduction

Let $E$ be an infinite compact subset of the complex plane $\mathbb{C}$ such that $\overline{\mathbb{C}} \setminus E$ is simply connected. There exists a unique exterior conformal representation $\Phi$ from $\overline{\mathbb{C}} \setminus E$ onto