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Methods for the computation of slowly convergent series and finite sums based on Gauss-Christoffel quadratures[†]

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Abstract

In this paper we give an account on summation/integration methods for the computation of slowly convergent series and finite sums. The methods are based on Gauss-Christoffel quadrature rules with respect to some nonclassical weight functions over $\mathbb{R}$ or $\mathbb{R}_+$. For constructing such rules we use the recent progress in symbolic computation and variable-precision arithmetic, as well as our MATHEMATICA package `OrthogonalPolynomials` [4, 26]. Together with our own results, we are also taking into consideration the use of other known results, especially the classical summation formulae of Euler-Maclaurin and Abel-Plana, in order to apply them afterwards in the computational techniques that we have developed recently. We present the Laplace transform method [15] and the contour integration method [21, 23], and give several numerical examples in order to illustrate the efficiency of different summation/integration methods.

Keywords: Summation, Gaussian quadratures, weight function, three-term recurrence relation, convergence, Laplace transform, contour integration.

MSC: Primary 65B10; Secondary 40A25, 41A55, 65B15, 65D30, 65D32, 30E20, 33C90.

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