Asymptotic expansion of generalized Durrmeyer sampling type series†

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Abstract

In this paper we consider a new definition of generalized sampling type series using an approach introduced by Durrmeyer for the Bernstein polynomials. We establish an asymptotic formula for functions \( f \) with a polynomial growth and as a consequence we obtain a Voronovskaja type formula. Then we consider suitable linear combinations that provide a better order of approximation. Finally, some examples are given, in particular certain central B-splines are discussed.

Keywords: Voronovskaja-type formula, moments, generalized Durrmeyer sampling series, linear combinations, splines.

MSC: Primary 41A25; Secondary 41A60, 94A20.

§1. Introduction

The classical Bernstein polynomials

\[
(B_n f)(x) := \sum_{k=0}^{n} p_{n,k}(x) f \left( \frac{k}{n} \right), \quad p_{n,k}(x) = \binom{n}{k} x^k (1-x)^{n-k} \quad (x \in [0, 1]),
\]

are known to give the most elegant proof of the Weierstrass approximation theorem by algebraic polynomials in the space of the continuous functions over the interval \([0, 1]\) (see

†Dedicated to Professor Paul Sablonnière on his 65th Birthday