



ISSN: 1889-3066

© 2014 Universidad de Jaén

Web site: [jja.ujaen.es](http://jja.ujaen.es)

Jaen J. Approx. 6(2) (2014), 219–231

**Jaen Journal**  
**on Approximation**

# Faber polynomials of matrices for non-convex sets<sup>†</sup>

Bernhard Beckermann and Michel Crouzeix

## Abstract

It has been recently shown that  $\|F_n(A)\| \leq 2$ , where  $A$  is a linear continuous operator acting on a Hilbert space and  $F_n$  is the Faber polynomial of degree  $n$  corresponding to some convex compact  $E \subset \mathbb{C}$  containing the numerical range of  $A$ . Such an inequality is useful in numerical linear algebra, it allows for instance to derive error bounds for Krylov subspace methods. In the present paper we extend this result to not necessarily convex sets  $E$ .

**Keywords:** GMRES, Krylov subspace methods, numerical range, Faber polynomials, polynomials of a matrix.

**MSC:** Primary 47A12; Secondary 65F10.

## §1. Introduction and statement of the main results

Consider a bounded operator  $A$  on a Hilbert space  $\mathcal{H}$  with spectrum  $\sigma(A)$ , for example a square matrix  $A \in \mathbb{C}^{N \times N}$ , and denote by  $\mathbb{P}_n$  the space of polynomials of degree  $\leq n$ . Following [18, 24], we are interested in giving upper bounds for the quantity

$$\delta_n(A) = \min\{\|p(A)\| : p \in \mathbb{P}_n, p(0) = 1\}, \quad n = 1, 2, 3, \dots, \quad (1.1)$$

<sup>†</sup>Pour Paul Sablonnière, à l'occasion de son soixante-cinquième anniversaire.

**Communicated by**  
A. Kroó

**Received**  
October 4, 2013  
**Accepted**  
December 5, 2013