ISSN: 1889-3066

Web site: jja.ujaen.es

Jaen J. Approx. 7(1) (2015), 11–56

Jaen Journal on Approximation

Weighted γ-K-functional and γ-modulus of smoothness on the semiaxis

Zoltán Markó

Communicated by
H. N. Mhaskar

Received
February 10, 2014
Accepted
October 11, 2014

Abstract

In this paper we investigate the γ-relative differentiation with the motivation of amending the order of the weighted polynomial approximation on the semiaxis for certain functions. By means of these tools, we give some definitions of generalized Sobolev spaces, K-functionals and moduli of smoothness. We estimate every one in terms of the others and, in the case of first order, we prove the equivalence. We show some possible applications and also other generalizations.

Keywords: γ-relative differentiation, weighted K-functional, weighted modulus of smoothness, Laguerre-weight.

MSC: Primary 26A15, 26A24, 41A17; Secondary 41A10.

§1. Introduction

The polynomial approximation is a traditional area in approximation theory. In the course of this, one might ask the question of the order of approximation. If X is a Banach space of functions with finite domains of $\mathbb{R}$, and P_n is the space of polynomials of degree at most n on X, then the error of the best polynomial approximation for a given function $f \in X$ is

$$E_n(f) := \inf_{p_n \in P_n} \|f - p_n\|.$$