On rational approximation and the discrete Hankel operator

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Abstract

In this paper we investigate questions related to rational approximation of functions $f$ given on a finite set of points on the complex plane. We define the discrete Hankel operator $B_f$ and investigate some of its properties. A theorem establishing a connection between the singular numbers of the discrete Hankel operator $B_f$ and the errors in best rational approximation of $f$ is proved. This result is an analogue of the Adamyan-Arov-Kreĭn theorem related to the theory of the Hardy spaces of analytic functions.

Keywords: rational approximation, the AAK theorem, Hankel operator, singular value.

MSC: Primary 30E10; Secondary 47B35, 41A20.

§1. Introduction

1.1. Notations

Let $K$ be a compact set in the complex plane $\mathbb{C}$. For any continuous function $g$ on $K$ denote by $\|g\|_K$ the norm of $g$ in the uniform metric on $K$,

$$\|g\|_K = \max_{z \in K} |g(z)|.$$