



# Regions of convergence for rational function series<sup>†</sup>

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## Abstract

We will define a bivariate logarithmic potential for balanced Borel finite signed measures on  $\overline{\mathbb{C}}$  and then we use it to characterize the regions of convergence of a rational function series in terms of certain limiting behaviors of the poles and zeros of its terms.

**Keywords:** regions of convergence, rational function series, rational approximation, bivariate logarithmic potential, logarithmic potential.

**MSC:** Primary 30E10, 40A30, 40-02; Secondary 30B10, 30E15, 31A05.

## §1. Introduction

Consider a function series

$$\sum_{k=1}^{\infty} r_{n_k}(z) \tag{1.1}$$

where each term  $r_{n_k}$  is a rational function with precisely  $n_k$  poles and  $n_k$  zeros on the extended complex plane  $\overline{\mathbb{C}}$ , counting multiplicities. Here, we assume that the sequence  $\{n_k\}_k$  is a nondecreasing sequence. The limiting behaviors of the poles and zeros of  $r_{n_k}$

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