Szász-Mirakjan-Durrmeyer and Baskakov-Durrmeyer operators with respect to arbitrary measure†

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Abstract

In this paper, we introduce Szász-Mirakjan-Durrmeyer operators and Baskakov-Durrmeyer operators with respect to an arbitrary measure, thus extending this natural generalization from the already studied case of the Bernstein-Durrmeyer operators to these two closely related cases. We establish convergence of the new operators, namely, pointwise convergence at each point of continuity of a function in the support of the measure, and uniform convergence in every compact set in the interior of the support of the measure. In case when the measure is finite, we also show convergence in the corresponding weighted $L^p$-spaces, $1 \leq p < \infty$. In the latter case, we also give estimates for the rate of convergence in terms of a K-functional.

Keywords: Szász-Mirakjan-Durrmeyer operator, Baskakov-Durrmeyer operator, uniform convergence, pointwise convergence, $L^p$-convergence.

MSC: 41A36.

†This work was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, under grant No. 20-130/1433 HiCi. The authors, therefore, acknowledge technical and financial support of KAU.