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A note on tensor product spline approximation via extension operators

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Abstract

In this note we look at anisotropic approximation of smooth functions on bounded domains with tensor product splines. The main idea is to extend such functions and then use known approximation techniques on $\mathbb{R}^d$. We prove an error estimate for domains for which bounded extension operators exist. This obvious approach has some limitations. It is not applicable without restrictions on the chosen coordinate degree even if the domain is as simple as the unit disk. Further for approximation on $\mathbb{R}^d$ there are error estimates in which the grid widths and directional derivatives are paired in an interesting way. It seems impossible to maintain this property using extension operators.

Keywords: anisotropic, approximation, splines, extensions.

MSC: Primary 41A15; Secondary 41A25, 46E35.

Recently, approximation using tensor product splines with different grid width and order in different coordinate directions, called anisotropic approximation, has gained some interest. For example in isogeometric analysis B-splines are used for finite element applications on bounded domains. The main reason for the usage of B-splines are their good approximation properties. While approximation theory for splines defined on d-variate boxes is well understood, there seems to be a certain gap concerning arbitrary domains. In the literature there are several results on anisotropic approximation (e.g., [1, 6, 7, 9]), but in all cases, there are more or less severe restrictions on the geometry of the considered domains. On the other hand, there is a large machinery for extending