On Montessus de Ballore’s theorem for nonlinear Padé-orthogonal approximants†

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Abstract

A convergence theorem for row sequences of nonlinear Padé-orthogonal approximants corresponding to a measure supported on a general compact subset of the complex plane is established. Our result is one of the natural extensions of the theorem of Montessus de Ballore for row sequences of classical Padé approximants.

Keywords: nonlinear Padé approximants of orthogonal expansions, Padé-orthogonal approximants, Fourier-Padé approximants, orthogonal polynomials, Montessus de Ballore’s theorem.

MSC: Primary 30E10, 41A21; Secondary 41A25.

§1. Introduction

Let $E$ be a compact subset of the complex plane $\mathbb{C}$ which contains infinitely many points such that $\overline{\mathbb{C}} \setminus E$ is simply connected. There exists a unique exterior conformal representation $\Phi$ from $\overline{\mathbb{C}} \setminus E$ onto $\overline{\mathbb{C}} \setminus \{w: |w| \leq 1\}$ satisfying $\Phi(\infty) = \infty$ and $\Phi'(\infty) > 0$. Furthermore, we assume that $E$ is such that the inverse function $\Psi = \Phi^{-1}$ can be extended continuously to $\overline{\mathbb{C}} \setminus \{w: |w| < 1\}$ (the closure of a bounded Jordan region and a closed finite interval satisfy the above conditions).

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