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On Loewner's characterization of polynomials

José M. Almira

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F. Marcellán

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Abstract

We prove a conjecture posed by Laird and McCann in 1984, providing a new demonstration of Loewner's characterization of polynomials.

Keywords: polynomials, exponential polynomials, functional equations, schwartz distributions, Montel type theorem.

MSC: Primary 26C05; Secondary 39B22, 39B32, 39A70.

§1. Introduction

We give a new proof of the following result.

Theorem 1.1 (Loewner, 1959). *Assume that* $d > 1$ *is a natural number. Let* $f \in C(\mathbb{R}^d)$ *and let* $R_f = \mathbf{span}\{f(L(x)) : L \text{ is an isometry of } \mathbb{R}^d\}$. *Then* f *is an ordinary polynomial if and only if* $\dim R_f < \infty$.

Note that in this theorem the word "isometry" means "Euclidean isometry", i.e. an isometry of $\mathbb{R}^d$ is just a rigid movement of the Euclidean space. These operators are characterized as compositions of a translation and a linear isometry. The result was demonstrated by Loewner [11] using that exponential polynomials can be characterized as elements of finite dimensional translation invariant subspaces of $C(\mathbb{R}^d)$ (see also