



# Expected zeros of random orthogonal polynomials on the real line<sup>†</sup>

Igor E. Pritsker

## Abstract

We study the expected number of zeros for random linear combinations of orthogonal polynomials with respect to measures supported on the real line. The counting measures of zeros for these random polynomials converge weakly to the corresponding equilibrium measures from potential theory. We quantify this convergence and obtain asymptotic results on the expected number of zeros located in various sets of the plane. Random coefficients may be dependent and need not have identical distributions in our results.

**Keywords:** polynomials, random coefficients, expected number of zeros, uniform distribution, random orthogonal polynomials.

**MSC:** Primary 30C15; Secondary 30B20, 60B10.

## §1. Asymptotic equidistribution of zeros

Zeros of polynomials of the form  $P_n(z) = \sum_{k=0}^n A_k z^k$ , where  $\{A_k\}_{k=0}^n$  are random coefficients, have been studied by Bloch and Pólya, Littlewood and Offord, Erdős and Offord, Kac, Rice, Hammersley, Shparo and Shur, Arnold, and many other authors. The early

---

<sup>†</sup>Research was partially supported by the National Security Agency (grant H98230-15-1-0229) and by the American Institute of Mathematics.

**Communicated by**  
G. López Lagomasino

**Received**  
August 26, 2016  
**Accepted**  
April 25, 2017