



On Hilbert lemniscate theorem for a system of quasidisks

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Abstract

Let K be a compact set in the complex plane consisting of a finite number of quasidisks. We study the rate of approximation of K from the outside by lemniscates in terms of level lines of the Green function for the complement of K .

Keywords: Hilbert's theorem, Green's function, equilibrium measure, quasi-conformal curve, lemniscate.

MSC: 30A10, 30C10, 30C62, 30E10.

§1. Introduction and main result

Let $K \subset \mathbb{C}$ be a compact set in the complex plane \mathbb{C} consisting of disjoint closed connected sets (continua) $K^j, j = 1, 2, \dots, \nu$, i.e.,

$$K = \cup_{j=1}^{\nu} K^j; \quad K^j \cap K^k = \emptyset \text{ for } j \neq k; \quad \text{diam}(K^j) > 0;$$

where $\text{diam}(S)$ is the diameter of $S \subset \mathbb{C}$. We always assume that $\Omega := \overline{\mathbb{C}} \setminus K$ is connected. Here, $\overline{\mathbb{C}} := \mathbb{C} \cup \{\infty\}$ is the extended complex plane.

According to the Hilbert lemniscate theorem (see [11, p. 159]), for any open neighborhood U of K , there exists a polynomial $p(z)$ such that

$$\frac{|p(z)|}{\|p\|_K} > 1, \quad z \in \mathbb{C} \setminus U, \quad (1.1)$$

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