On the degree of rational approximation of Markov functions on discrete sets

Vasily A. Prokhorov

Dedicated to the memory of my dear teacher, Professor Valery V. Vavilov.

Abstract

This article is devoted to results of rational approximation of the Markov function

\[ \hat{\alpha}(z) = \int_{F} \frac{d\alpha(x)}{z - x}, \]

where \( \alpha \) is a positive Borel measure with support \( \text{supp} \alpha = F = [a, b] \subset (0, \infty) \) and \( d\alpha/dx > 0 \) a.e. on \( F \) (with respect to the Lebesgue measure). We study asymptotic properties of the best uniform rational approximation of Markov functions \( \hat{\alpha} \) on point systems \( E_N \subset (-\infty, 0) \) when the number of points \( N \) in the set \( E_N \) and the degree of rational approximants \( n \) satisfy an asymptotic relation \( N/n \to \theta > 2 \) as \( n \to \infty \). The degree of rational approximation is described in terms of the solutions of certain logarithmic potential-theoretic problems, central among which is a minimal energy problem in the presence of an external field. We also investigate the limit distribution of poles of the best rational approximants and of points of Chebyshev alternance.

Keywords: rational approximation, Markov function, degree of approximation, approximation on discrete sets, Hankel operator, potential theory.

MSC: Primary 30E10; Secondary 41A20, 41A25.