Quasi-interpolation for near-boundary approximations

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Abstract

In this work we address the problem of approximating a smooth function on a bounded domain from a set of data points on which we know the values of the objective function. While we can generally guarantee impressive approximations in the interior of the domain, the theory does not extend to the boundary of the domain. Indeed, numerical experiments present all forms of artifacts when performing approximations near the boundary of the domain. To achieve adequate approximations near boundaries, we will build upon our previous work, in which we have managed to construct high-order approximations to singular functions. By considering the boundary of the domain as a singularity, we show that we can similarly return high-order approximations to the objective function, even in the immediate vicinity of the boundary of the domain.

Keywords: quasi-interpolation, multivariate functions, moving least squares.

MSC: Primary 41A05, 41A63; Secondary 65D05.

§1. Introduction

Consider the problem of approximating a smooth function on a bounded domain from a set of data points on which we know the values of the objective function. While there is an extensive theory which guarantees impressive results in the interior of the domain, where we would generally have adequate input information, it is unfortunate