V. Markov’s problem for $k$-absolutely monotone polynomials and applications†

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Abstract

We consider the classical problem of maximizing the value of the derivative of a polynomial at a given point $x_0 \in [-1, 1]$. The corresponding extremal problem for general polynomials in the uniform norm was solved by V. Markov. In this paper, we consider the analog of this problem for $k$-absolutely monotone polynomials. As a consequence, we solve the analog of V. Markov’s problem, find the exact constant in Bernstein’s inequality and give a new proof of A. Markov’s inequality for monotone polynomials.

Keywords: Markov’s inequality, Nikolskii inequality, $k$-absolutely monotone polynomials, shape-preserving approximation.

MSC: Primary 41A17; Secondary 41A29, 41A44.

§1. Introduction

The purpose of this paper is to give a short proof of a significantly more general result than the one appeared in an unpublished manuscript of the first author [8]. Perhaps slightly surprisingly, the results of the present paper found applications in theoretical computer science. Y. Filmus, H. Hatami, N. Keller and N. Lifshitz [4, Section 3.4] and

†O.K. greatly acknowledges Mathematical Science Research Institute (Berkeley) and Stanford University for excellent working conditions and financial support. S.Z. is grateful to incoming mobility program of Bourgogne-Franche-Comté for the financial support.