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# Approximation of functions of several variables by continuous linear splines on rectilinear grids<sup>†</sup>

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## Abstract

We consider approximation of functions of several variables by continuous linear splines interpolating the given function in the knots of a rectilinear lattice. For function classes defined in terms of a modulus of continuity, we give an exact estimate for the error of approximation. In the particular case when the modulus of continuity is concave and the distance between points in  $\mathbb{R}^d$  is measured in the  $\ell_p$ -norm with  $1 \leq p \leq 3$ , we calculate an explicit value of the exact approximation error on the class. Surprisingly, the behavior changes dramatically if  $p > 3$ . We show that the our estimate is no longer true, in general, when  $p > 3$ .

We also consider approximation of a first derivative of a function by the corresponding derivative of the linear continuous spline and obtain an upper estimate for the error of approximation for an arbitrary modulus of continuity, all  $1 \leq p \leq \infty$ , and triangulations of the staircase type.

**Keywords:** continuous linear splines, polyhedral functions, rectilinear lattice, modulus of continuity, simultaneous approximation.

**MSC:** Primary 41A15, 41A28; Secondary 41A63.

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