



About Lebesgue inequalities on the classes of generalized Poisson integrals[†]

A. S. Serdyuk and T. A. Stepanyuk

Abstract

We establish asymptotically best possible Lebesgue-type inequalities on the sets $C_{\beta}^{\alpha,r}$ of generalized Poisson integrals from continuous periodic functions. We prove that obtained estimates are also asymptotically exact on some important subsets $C_{\beta}^{\alpha,r}(\varepsilon)$.

Keywords: Lebesgue inequalities, Fourier sums, classes of convolutions of periodic functions, best approximation.

MSC: Primary 42A10, 42A05, 41A16; Secondary 41A50.

§1. Introduction

Let L_p , $1 \leq p < \infty$, be the space of 2π -periodic functions f summable to the power p on $[0, 2\pi)$, in which the norm is given by the formula $\|f\|_p = \left(\int_0^{2\pi} |f(t)|^p dt \right)^{\frac{1}{p}}$; L_{∞} be the space of measurable and essentially bounded 2π -periodic functions f with the norm $\|f\|_{\infty} = \operatorname{ess\,sup}_t |f(t)|$; C be the space of continuous 2π -periodic functions f , in which the norm is specified by the equality $\|f\|_C = \max_t |f(t)|$.

[†]This work is partially supported by the Volkswagen Foundation (VolkswagenStiftung), program “From Modeling and Analysis to Approximation”. The second author is supported by the Alexander von Humboldt Foundation.

Communicated by
F. Dai

Received
May 28, 2020
Accepted
July 13, 2020