ISSN: 1889-3066

Web site: jja.ujaen.es

Jaen J. Approx. 12 (2021), 89–109

Jaen Journal on Approximation

On some inequalities for sine polynomials

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Communicated by S. Tikhonov

Received February 23, 2021
Accepted April 8, 2021

Abstract

Inspired by the work of L. Fejér and some of his followers, we present inequalities for various sine polynomials. Among others, we offer a new extension of the classical Fejér-Jackson inequality and we prove that the inequality

$$\sum_{k=1}^{n} \binom{n-k+c}{n-k}^{-1} \sin(kx) < 0 \quad (c \in \mathbb{R} \setminus \{-1,-2,\ldots\})$$

holds for all $n \geq 2$ and $x \in (0,\pi)$ if and only if $c \in [-3/2, 4/3]$. As an application of this result we obtain that the function

$$x \mapsto x\Big(1 - \frac{{}_2F_1(1,1;a;x)}{x^2+bx+1}\Big) \quad (-1/2 \leq a \leq -1/3;\ -2 < b < 2)$$

is absolutely monotonic on $(0,1)$. Here, ${}_2F_1$ denotes the Gaussian hypergeometric function.

Keywords: sine polynomials, inequalities, absolutely monotonic, hypergeometric function.

MSC: 26A48, 26D05, 26D15, 33B10, 33C05, 41A44.

§1. Introduction

I. In 1910, Fejér conjectured that all partial sums of the Fourier series

$$\sum_{k=1}^{\infty} \frac{\sin(kx)}{k} = \frac{\pi - x}{2} \quad (0 < x < \pi)$$