Departamento de Economía Aplicada

DOCUMENTOS DE TRABAJO

UNIVERSIDAD DE JAÉN

URBAN MICROECONOMICS WITHOUT MUTH-MILL: A NEW THEORETICAL FRAME

Javier Rodero Cosano*; Pablo Brañas Garza* and

Inmaculada Fernández Piñar*

WP 9905/Nº 5

Dirección para comentarios y críticas:

Dpto. de Economía Aplicada
Universidad de Jaén
Campus Universidad de Jaén
Paraje de Las Lagunillas s/n
23071 Jaén

* Universidad de Jaén. Área de Fundamentos del Análisis Económico (Departamento de Economía Aplicada).

URBAN MICROECONOMICS WITHOUT MUTH-MILL: A NEW THEORETICAL FRAME

Javier Rodero Cosano*
Pablo Brañas Garza*
Inmaculada Fernández Piñar*

J.E.L. Classification numbers: D21, L74, R21, R31.

Abstract

In this paper we develop an alternative urban model in a linear way. We consider a city with an industrial area localised in the edge, it creates a negative externality that affects to the closer neighbourhoods, creating an asymmetry in the housing demand. Also, there are heterogeneous agents, skilled workers, unskilled ones and landlords. Each one has a different utility function and working place (the last one doesn't work). We conclude that the city presents an asymmetric distribution of population density and prices. All this work is done removing the standard monocentric model hypothesis.

Keywords: urban economics, asymmetry, externalities, amenities.

* Universidad de Jaén. Área de Fundamentos del Análisis Económico (Departamento de Economía Aplicada).

INTRODUCTION

The study of the cities from a economic perspective places the *urban location theory,* in the microeconomic discipline. The developed literature in this field has led to the formation of two clearly opposite currents of thought: the orthodox one and the alternative one.

The principal core of the neoclassic urban location theory is made by the standard monocentric model or dilemma model, that was developed by Muth (1969) and Mills (1967). This model central idea, under the hypothesis that the housing is cheaper on the city outskirts (corner), is that individuals with given incomes decide their residential place according to proximity to the C.B.D.[1] —the monocentric city is supposed. Considering the increasing commuting cost to the C.B.D. (commuting cost includes monetary and time expenses) and the lower housing price on the corner.

In short, an individual will be prepared to pay a determinate price in return for his housing only according to the distance at the C.B.D., so it is the unique explanatory variable. This would be the known as *compensation space/access hypothesis.*

In this sense, we could say that Muth-Mills´ has two basic characteristics. The first, the C.B.D. of the city is simply a point that exerts a force of inertia towards itself and rules the urban structure (spatial symmetry) and, second, the distance to the C.B.D. is the most important variable that determinates the housing price.

Exactly, the successive criticisms that the monocentric model has been receiving are related to its basic assumptions, in other words:

a) The distance to the C.B.D. is one exogenous variable among many ones that explain the housing demand (Wilkinson & Archer, 1973) and, therefore, those will be also considered.

b) The cities doesn't present a structure of simple centre, they have became decentralised, creating new alternative important cores (Turnbull, 1990).

The heterodox current is perfectly contained in the model of Tiebout (1956). His fundamental idea is: individuals which are preparing to buy a house, establish their preferences considering the characteristics of the place and the environment (apart from the accessibility degree to C.B.D.) for example: environmental attributes, neighbourhood quality, social status, etc. Therefore, the housing is recognised as a markedly heterogeneous good.

The development of the cities has evolved towards a certain magnitude structure, that takes place a very important advance in the urban literature with alternative models, successive extensions of the already existing and numerous empirical studies. These remove the restrictive assumptions of the monocentric model, so the existence of a single core is not considered, because of the economic development new activity cores are being created that help the decentralisation of the city. In this sense, the studies of Henderson (1985) and the Turnbull ones (1990) are very important, because they have developed the concept of *multicentric structures.*

---

[1] The C.B.D. (Central Business District) is considered as the place where the commercial and employment centres are localised.

In the other hand, geographic economy studies the localisation of the productive activity. That discipline tries to discover why the activity tends to concentrate itself in a small number of cores or cities (Fujita, M. & Thisse, J.F., 1996). For that, the models establish a partial equilibrium of the economic activity with two types of forces: centripetal (or agglomeration) and centrifugal (or dispersion), both these push and attract consumers and factories until they obtain an optimum localisation. The agglomeration economics are considered the principal institutions where technologic and social innovations are developed with the market and nonmarket interactions.

Therefore, a model of city that explains the localisation of individuals must include the exerted influence over the city through the localisation of economic activity. In this sense, the so called *marshallians externalities* (Marshall, 1890, 1920) take a great relevance. They suppose a centripetal force makes economic agents tend to get more crowded together every time in certain places, because some factors cause a greater diversity and higher specialisation in the productive processes and, therefore, major variety of consumption goods.

The installation of new factories in those regions creates new incentives to attract workers that look for better employment and wages. And so, a very attractive place is built for factories they hope to find qualified personal, speciality services and new points of sale for their goods; the individuals will tend to be localised close his job place.

Since the concept of agglomeration can take diverse meanings (Fujita, M. y Thisse, J.F., 1996), in this paper we introduce a model of lineal city that present two type ones fundamentally. An agglomeration created by the existence of factories outside city. These influence strongly over residential localisation of the individuals working there. The other agglomeration is derived from the economic activity of the C.B.D. and determinates the localisation decisions of the rest of individuals. Considering this situation we present an alternative model of city respect at the Muth-Mills´ where urban localisation choice is asymmetric.

1. A FIRST INSIGHT IN THE MODEL

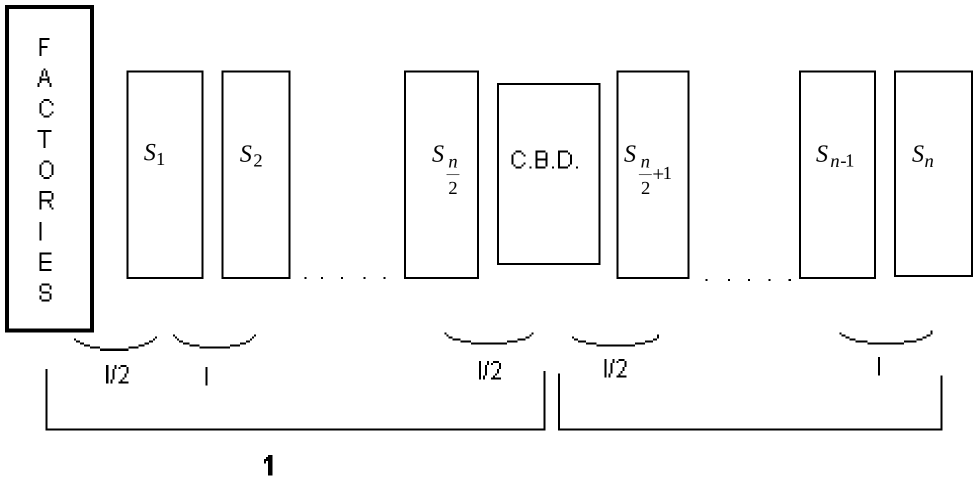
1. 1. Physic Structure of the City

The real structure of the city is bi-dimensional, but the economic analysis requires only one dimension because the distance determines all goods features. Being more accurate, using the Euclidean distance from each element central point to another element central point (for example, the centre of the city or one city suburb) and its direction (north-south), we can define the first element position. That is the reason why we say the city is lineal, although not strictly uni-dimensional.

1.1.1. City elements

This elements are restrictions (institutions) for the model. Those parameters could be variables in long term, but their characterisation are fixed in this initial model.

Graph 1: the city elements



The elements are the next: C.B.D. (there is only one), suburbs (where people lives), and factories.

1.1.1.1. C.B.D., Central Business District

It is located at the city centre (approx.) and, by hypothesis, there is not supply of dwellings there. The whole surface is used as a production factor for two purposes: the production of one good, at the moment called organisation; and the production of another one named "housing rent". The first is used by the factories and the second by the "housing renters".

At the moment, we suppose that the owners of the C.B.D. surface are living abroad (absent landlords), and the price is determined in a competitive market, where factories and renters demand surface.

1.1.1.2. Suburbs

There are *n* residential zones, also called suburbs. Inside the suburbs the lots can be utilised as homes or shops. The economic activity inside the suburb is related with the distribution good.

1.1.1.3. Factories

They are the physic zones where the industrial firms manufacture the good. The good will be sold in a global market (international). The owners of this land are the own managers. The surface each factory spends will be adjusted at the long term.

Our focus is quite different respect the previous ones in urban theory, because the asymmetry introduced at the model (the factories and their externality). If we begin with a linear city where there are not factories, in one moment of time, the first factory should decide where it would come into. The city planners (or the market) an they know the externality the factory causes on the population (pollution, noise...), then they will decide that the better location for the industrial firms is the city corner. By convention it will be named South.

If the externality is only a disamenity for the individuals, not for another factories, it is easy to suppose that the next managers (other factories) will locate close to the first one, where the land is cheaper (there is not home use) than in another city place. Under this perspective is possible to define the genesis of the industrial zone. Then, we will name factory (*fab*) the whole industrial zone for the rest of the paper.

1.1.2. Distances between elements

As is defined in the graphic shown before (g.1), the distance between *C.B.D.* and the factory has been normalised to 1. Let us denote l to the distance between two suburbs; $l/2$ to the distance between *C.B.D.* and the first suburb and $l/2$ between *fab* and the closed suburb. The distances are taken between central points of the elements, then it is easy see that:

$$1=\left(\frac{n}{2}-1\right)l+\frac{l}{2}+\frac{l}{2}=\frac{n}{2}l \quad \Rightarrow l=\frac{2}{n} \quad \Rightarrow \frac{l}{2}=\frac{1}{n} \qquad [1]$$

The distance between one suburb S_i and the *C.B.D.* is called l_i, where:

$$l_i = d(S_i,CBD)=\left|\frac{2i-(n+1)}{n}\right|=\begin{cases}\dfrac{(n+1)-2i}{n} & i\le\dfrac{n}{2}\\[2ex] \dfrac{2i-(n+1)}{n} & i>\dfrac{n}{2}\end{cases} \qquad [2]$$

$$1\le i\le n$$

And the distance between the factory and the suburb i, is named m_i, where:

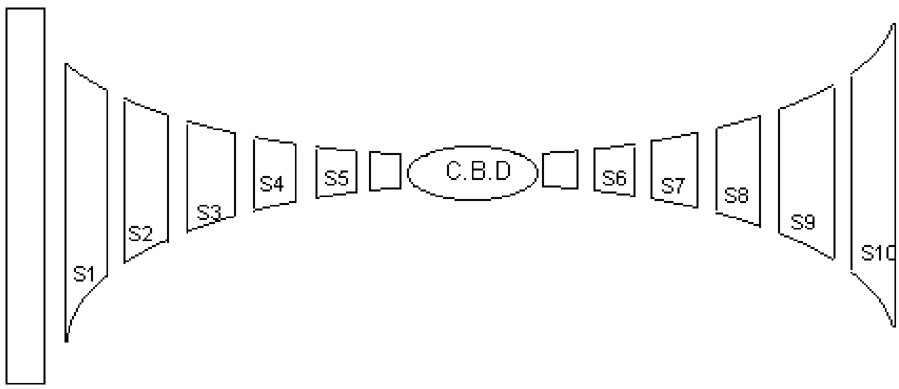
$$m_i = d(S_i,fab)=\frac{2i-1}{n} \qquad [3]$$

$$1\le i\le n$$

1.1.3. Surface of the elements

The surface of each element will depend of the kind of model of city we use. Under our focus the city is linear, so the elements surface is defined by their "deep". Although, we use an semicircular linealised structure in what the city is divided in both semicircles. Between both we can find a clear asymmetry, because at the south there is a factory (externalities), not at the other direction: by hypothesis called North. The only communication between North and South is through the C.B.D.. All these assumptions allow to approach our city as an linealised model, as it is shown below.

Graph 2. The semicircular city



It is easy to prove that, under our assumptions, the surface of each suburb is:

$$S_i = C_1 |n - 2i + 1| \qquad 1 \le i \le n \qquad [4]$$

The constant C_1 is depending of the ring portion each suburb use. Not all the ring spaces are allowed to be occupied by the dwellings. We suppose there are free spaces between rings where highways, parks, and other public infrastructures are allocated. Being more accurate, we can show that $C_1 = \pi l o$, where *2o* is the real surface each suburb takes of space (*0<2o<1*).

The C.B.D. surface will be named *CB*, and the factory's will be *FAB.*

1. 2. Individuals endowments

In the model we analyse three types of individuals. All are defined by their production factors endowment, being *I* the whole number of individuals living at the city (is possible introduce a fourth individual: external capitalist in the global money market, but it is not relevant at the moment).

Unskilled workers (UW). Their endowment is an unit of not qualified labour, that they supply in inelastic way. They do not save, nor have any kind of wealth (at least in the static focus). There are I_1 UW individuals in the city, and not one else supplies unskilled labour.

Skilled workers (SW), having an unit of qualified work, also supplied in inelastic way, without any save or wealth. In the city there are I_2 SW, but there are many other living abroad who are supplying skilled labour too (we suppose there is perfect mobility for skilled labour).

Landlords, have K units of capital for investment. They do not work. They have wealth and they got their income from capital returns (investment at the city or away). They can rent some of their properties to the citizens, this activity is defined as "real state agency", but is imposed that this work does not give them any desutility. There are I_3 landlords living in the city.

Obviously:

$$I \equiv I_1 + I_2 + I_3 \qquad [5]$$

The labour market of unskilled workers is closed, there is not supply/demand from external citizens. The only competition is among internal individuals. The wage they get is w_1. Anyway in this framework the assumption of a open market doesn't make much difference.

The skilled market is quite different, because is open. In this way, we admit the possibility that external citizens came to this city if they find better wage or other. Is not false that skilled are more mobile than unskilled, although the commuting cost -for externals- are not taken into account.

The capital market is absolutely free in a market under perfect competition in the city, although there are some restriction for the building sector.

1.3. Firms

We consider three types of enterprises depending the kind of product they sell in the city market.

1.3.1. Industrial firms

These firms produce a consumption good, x. And they are located in the border of the city, because of their externality. The good is considered homogeneous at world markets, so it is traded under perfect competition. The demand of x is given exogenously.

The firm use two types of installation for carry out x: the factory at the border, using unskilled workers (L_{UW}); other at the C.B.D. for administrative procedures, using D units of physic space of C.B.D. and skilled workers (L_{SW}).

We suppose Cobb-Douglas constant returns technology, and because there is perfect competition, the price is given (normalised to 1). The number of firms is indeterminate. The production function is,

$$x = L_{UW}{}^{\alpha} L_{SW}{}^{\beta} D^{1-\alpha-\beta} \qquad [6]$$

The factory produces an externality over the residential suburbs, P_i, (also called disamenity). It will be commented later.

1.3.2. Commercial firms

There are some retailers who are selling the good x at the suburbs. This family of goods, called y_i, is supplied in perfect competition (freedom of entrance, and so for). The goods are perfect substitutes for the consumers (not in perfect proportion, 1:1, because there are some cost of removal from the place where the retailers are situated to other places).

For the production of y_i is necessary, besides x (called x_i): physical space in the suburb (F_i). The production function present constant returns, following the next form,

$$y_i = min\{x_i, \delta F_i\} \qquad [7]$$

$$\delta > 0$$

The number of firms, as before, is not relevant, the only important are the results at aggregate level by suburb. There are n markets of x, where this good is transformed and sold as y_i. The price of x is 1 and the price of y_i is b_i.

1.3.3. Houses builders and real state agencies

There are two kinds of firms related with the supply of dwellings: ones, who are building the homes, others selling the services (renting). This topic will be detailed later.

1.4. Housing supply

The building sector, under our hypothesis, is a sequential play that is solved in some iterations (steps): home built; the real state agencies buy the dwellings; homes are rented to individuals and commercial firms.

Step 1. Home built

Some firms built the houses before. Houses are homogeneous in attributes in each suburb, not outside. The firms live under perfect competition. Their decision variable, H_i, is the number of houses to build in each suburb. Their cost function is,

$$C = f(H_i, S_i) = C(H_i) \qquad [8]$$

The firms sell the houses to real state agencies (remember that it is an activity performed without desutility by the landlords) at the price a_i, using a bargaining procedure that will be analysed below.

Step 2. The real state agencies (RSA) buy the dwellings and rented to individuals and commercial firms.

The building firms sell the whole suburbs to RSA. The suburbs are acquired by landlords, even syndicates of them. The procedure is sequential: beginning at the last suburb and finishing at the first (closest to the C.B.D.)

After the bargain, the new owners -RSA or landlords- rent some houses to the citizens (h_i) and commercial firm (F_i) who are biding for the dwellings. The commercial firms use their houses (F_i) as input of their production function [7]. By hypothesis, the market is cleared, then,

$$H_i = h_i + F_i \qquad [9]$$

It is important to denote that there are differences if landlord (RSA) are citizens or live abroad. If RSA lives abroad, he has some commuting cost every day he goes to the city for management his business, even uncertainty about how the tenants will look after their properties.

Because the landlords citizens can pay more in the bargain than the external, then it seems logic suppose that the renters -if capital market is perfect- are always citizens. We defined before, that landlord do not get any desutility managing their properties, at least that will be the same that managing other portfolio (for example, shares), then the management is just a constant in their utility function.

The syndicate of RSA will supply the dwellings under monopolistic competition. Houses are perfectly homogeneous inside the suburb, it is not possible to find different prices at the same zone.

The rent housing price that the tenant have to pay will be a_{1i}, the demand per suburb will produce the level of occupation at that price. The number of leased houses, at i suburb, will be h_i,. So the level of congestion is,

$$Congestion\ \ Index \rightarrow CI_i = \frac{h_i}{S_i} \qquad [10]$$

In each suburb i there are living $I_{i,j}$ individuals of class j, $i=1,2,.....,n$, $j=1,2,3$. Obviously,

$$I_j = \sum_{i=1}^{n} I_{i,j} \qquad \forall\ j = 1,2,3 \qquad [11]$$

$$h_i = \sum_{j=1}^{3} I_{i,j} \qquad \forall\ i = 1\_n \qquad [12]$$

1.5. Industrial Externality

As was explained before, the industrial firms produce a negative externality over the residential suburbs P_i. The disamenity is bigger if the suburb is closer to the factory.

The rationalisation of this phenomenon is simple, based not only in pollution effects, also crime, noise, etc.

We suppose that the externality is linearly decreasing from the border (factory) until a maximum distance equal to 1 (the C.B.D.), where the disamenity effects are nill. If we call P_i to this disamenity we have that:

$$\begin{cases} P_i = \mathrm{P}(m_i) = (1\text{-}\mathrm{m_i}) & m_i \leq 1 \\ P_i = \mathrm{P}(m_i) = 0 & other \end{cases} \qquad [13]$$

1.6. Government

There is an authority in the city (despotic) running under balanced budget. His procedures are mechanical but not neutral.

The income of the government is a linear tax over dwellings under a fixed rate, named *t*. The government has two types of expenses: general public services for the whole city (*R*), and some public expenses in infrastructure in a determined suburb (G_i). We suppose that G_i is linearly decreasing with the distance to the C.B.D.

$$G_i = (1 - l_i)C_3 \qquad [14]$$

This hypothesis could seen to be strong, although it is more realistic than others, as for example: G_i equal in each suburb. The government uses to play as centripetal force in the dynamic of cities. *R* will be the difference between income and local public service expenses (balanced budget).

$$\sum_{t=1}^{n} t \cdot a_{1i} h_i = R + \sum_{t=1}^{n} G_i \qquad [15]$$

s. t. $0 \leq C_{3,}$ $R \geq 0$

2. FURTHER INSIGH IN THE MODEL

2.1. Some price solutions

1) *Labour wages*. There are I_1 individuals that supply their unskilled work in an inelastic way. The whole supply is I_1, because there are not foreign people working on to the city. The wage will be fixed by the factories,

$$w_1 = \overline{w}_1 \qquad [16]$$

All the skilled labourers work at the C.B.D., earning an competitive international wage. The labour supply is also inelastic.

$$w_2 = w_2^* \quad [17]$$

where w_2^* is the international skilled competitive wage.

There is not unemployment, so

$$\begin{aligned} I_1 &= L_{uw} \\ I_2 &= L_{sw} \end{aligned} \quad [18]$$

2) *The price of the good* y_i. The commercial firms sell the generic good x at the suburbs, in quantity y_i. The production [7] is related with the space of lot[2] at the suburb i and the good x. Solving the Leontieff problem the amount of y_i is

$$y_i = x_i = \delta F_i \quad [19]$$

The profit function of commercial firms, in aggregate level, is

$$\Pi = b_i y_i - a_{1,i} F_i - x_i \quad [20]$$

where b_i is the price of y_i; $a_{1,i}$ is the housing price (renting price) at the suburb i; and x_i the price of x.

If x is a numerarie, then the profits function is,

$$\Pi = y_i \left(b_i - 1 - \frac{a_{1,i}}{\delta} \right) \quad [21]$$

In perfect competition, as the profits must be zero, the price of the good y at each suburb i, will be,

$$b_i = 1 + \frac{a_{1,i}}{\delta} \quad [22]$$

The price is related with the numerarie and the housing price in each suburb, when the dwelling price is higher the good y_i will be more expensive, and is independent of the quantity y_i

And, the lot surface utilised -demanded- by the commercial firms for the distribution will be,

$$F_i = \frac{y_i}{\delta} \quad [23]$$

3) *Landlords profits*. The syndicate or single landlords buy houses, at price a_i, to lend it to individuals and commercial firms. The total amount of dwelling they rent is h_i+F_i. Their profits function will be,

[2] We suppose there are different prices for the lots: $a_{c,i}$ for shops and $a_{1,i}$ for houses. But if there is perfect competition the price will be the same, because lots could be utilised at homes as well as shops, as we will suppose later.

$$\Pi_i^A = a_{1,i}(h_i + F_i) - a_i H_i - aIMV \qquad [24]$$

where $a_{1,i}$, is the renting price at each suburb i; a is the lot price at the C.B.D.; and a*IMV are the commuting cost for non-citizens landlords, being

$$IMV = \begin{cases} 0 & for\ residents \\ 1 & for\ others \end{cases}$$

because foreign landlords have some commuting cost in their housing management.

In equilibrium, h_i+F_i= H_i, as there are not empty houses. Under perfect competition, when the profits are zero for foreign landlords, the housing price is,

$$\begin{aligned} H_i(a_{1,i} - a_i) &= a \\ a_{1,i} &= \frac{a}{H_i} + a_i \end{aligned} \qquad [25]$$

where a is the lot price at the C.B.D., and a/H_i are the unit commuting costs that the foreign landlords have.

If the housing price, $a_{1,i}$, determined by the local landlord, is not very high, the foreign ones can not get into the city (their commuting cost are not covered, and their profits would be negative).

Then the landlords profits function - there are only residents - is

$$\Pi = a_{1,i}H_i - a_i H_i \qquad [26]$$

If we introduce [25] in [26], it is easy to prove that,

$$\Pi = H_i\left(\frac{a}{H_i}\right) + a_i - a_i \Rightarrow \Pi = a \qquad [27]$$

Where a is the limit profit the landlords get at each suburb, i.e. the lot price at the C.B.D. they have not to pay because they are residents. We suppose each landlord gets the same share of the market, so because there are n suburbs and I_3 landlords, the profit each one earns is,

$$\Pi_i = \frac{an}{I_3} \qquad [28]$$

Finally, the income of the landlord, w_3, will be:

$$w_3 = k(1+r) + \frac{an}{I_3} \qquad [29]$$

where k is his capital endowment.

We suppose, $w_1 < w_2 < w_3$.

4) *Lot prices*. The surface of the C.B.D. -*CB*- is utilised by the industrial firms as a production factor (*D*) -for administrative procedures-, and foreign landlords for management of their houses (*A*). If we suppose there are not foreign landlord, the surface will be

$$D=CB \tag{30}$$

And the price of lots at the C.B.D. will be positively related with the production of the good *x*, because industrial firms demand the surface, so

$$a = f(x) \tag{31}$$

if production increases, the price will be higher because the surface is limited.

We suppose that the housing price that the landlords pay to the builders, a_i, is the cost of building (under perfect competition). So, if the number of houses is bigger the cost -the price- will be lower; then, the relation between a_i and H_i will be negative (or non positive).

Finally, the housing price at each suburb, $a_{1,i}$, will be

$$a_{1,i} = \frac{a}{H_i} + a_i \quad \Rightarrow \quad a_{1,i} = \frac{a}{\underset{-}{H_i}} + a_i(\underset{-}{H_i}) \tag{32}$$

is related negatively with H_i. As both are decreasing - $\frac{a}{H_i}$ and $a_i(H_i)$ -

When there is not uncertainty, the landlord knows the amount of houses that are demanded at price $a_{1,i}$, then they know the value of h_i.

$$h_i = h(\underset{-}{a_{1,i}}) \tag{33}$$

The number of commercial firms -and the surface they utilised- is related with the population of the suburbs, number of individuals living in each suburb, h_i.

$$F_i = F_i(\underset{+}{h_i}) \tag{34}$$

If landlords know it, the builders of dwellings also have the same information, they do a perfect planning: the number of houses they built is $h_i + F_i$.

2.2. Consumer behaviour

We consider a set of goods, y, that are produced in the n suburbs, and two sets, z and v, composed by all the goods consumed in every suburb, so that, the quantity consumed of y_i is z_i:

The consumption pattern of the particular good y_i in suburb i can be characterised by the minimisation by every consumer of the expense on the good and of commuting costs that its buying implies. These will depend of the distance between the resident an buying zones. The corresponding expression would be:

$$\mathrm{i} = \mathrm{argmin}\left\{\mathrm{b_i} + CC^{*} d(i,k)\right\} \qquad [37]$$

$$i = k$$

We suppose an individual will buy in his suburb, because the commuting costs are enough elevated to buy it at other neighbourhoods. These costs are not compensated for the possible savings in prices. I.e. the production of the transport utility is more efficient if it is produced by the retailer than by the consumer.

The consumption of the j class is $z_{i,j}$ and $v_{i,j}$, being $z_{i,j}$ the consumption of y_i in the suburb i and $v_{i,j}$, the consumption of housing at the suburb i.

We use the following analytical expressions to define the demand function $z_{i,j}$:

$$z_{i,j} = \phi(v_{i,j}) \qquad [38]$$

$$z_{i,j} = z_j v_{i,j} \qquad [39]$$

So, the consumption of y_i will depend of the amount of v_i that the individual of the kind j consumes; that means that z_i will be uniformly distributed in the time.

In the other hand, we define the consumption of v as the time that the individual of the kind j lives in each suburb[3], so that:

$$v_j = (v_{1,j},....,v_{n,j}) \qquad [40]$$

With that we write:

$$I_{i,j} = I_j v_{i,j} \qquad [41]$$

$$\forall_{i,j}$$

it indicates that the number of individuals of the kind j with residence in the suburb i is equal at the number of individuals j that consume v_i.

[3] To be exact, we can define v_j^k, k=1_I$_j$ for every person of j class, or think in probabilistic terms.

Once we are at this point we will have to solve the problem of the consumer. This implies to maximise the utility of the individual derived of consumption of housing and good y_i. The function is defined,

$$U: \quad \Re_+^{2n} \quad \rightarrow \quad \Re$$

But the consumption decision set of the individual is reduced to a vector of n+1variables:

$$(v_{1,j},....,v_{n,j};z_j)$$

The utility function to maximise would be given by the following expression:

$$U_j(.) = \sum_{i=1}^{n} u_{i,j}(v_{i,j}) + u_{c,j}(z_j) \qquad [42]$$

$$\text{s.t.} \sum_{i=1}^{n} v_{i,j} = 1$$

$$v_{i,j} \geq 0$$

and a budgetary restriction that we subsequently going to specify.

The function [42] is additively separable, with the needed restrictions, these refer to that every individuals live in a zone during a certain time (non negative), the sum of all times will be equal to the duration of model, that is, one unit of time.

Also, we suppose that all individuals of the all kinds like the same the good y_i and, as we early marked, they don't consume either of it in other different suburb to i. Therefore we have that:

$$u_{c,j}(z_j) = u_c(z_j) \qquad [43]$$

Next we introduce external conditions that influence over the level of satisfaction of the consumer. The first, pollution exists and it depends of the suburb where individual lives and the time he stays on. It is a function as:

$$P^* = P^*(\underset{+}{P_i}, \underset{+}{v_{i,j}}) \quad ; \quad P^*(P_i, 0) = 0 = P^*(0, v_{i,j}) \qquad [44]$$

Of course, more consumption of v_i makes more intense the absorption of pollution. This will not exist when the individual stays on clean suburb or during a moment he wouldn't be in a suburb with pollution.

Second, as the local government makes a public expense in each suburb that affect more positively the consumer if more time is spent by the individual in that suburb, analytically this is:

$$G^* = G^*(G_i, v_{i,j}) \quad ; \quad G^*(G_i, 0) = 0 = G^*(0, v_{i,j}) \qquad [45]$$

+ +

The same as in the previous case, the public expense function will be increasing in v_i.

The third aspect that determinates the individual utility's is the externality produced by other individuals (the inferior groups) that live in other suburbs. So, we would define the following increasing functions:

$$V_1{*} = V_1{*}(v_{i,1}, v_{i,j}) \quad ; \quad V_1{*}(v_{i,1},0) = 0 = V_1{*}(0, v_{i,j}) \qquad [46]$$

+ +

$$V_2{*} = V_2{*}(v_{i,2}, v_{i,j}) \quad ; \quad V_2{*}(v_{i,2},0) = 0 = V_2{*}(0, v_{i,j}) \qquad [47]$$

+ +

In short, the level of pollution, public expense and the residence of other kind of consumers affect the individual utility.

Next, we going to detail the consumer problem for each kind of individuals.

1) *Unskilled workers*: this group of individuals will have to maximise the following function:

$$Max \quad \sum_i u_1\left(v_{i,1}, P{*}(P_i, v_{i,1}), G{*}(G_i, v_{i,1})\right) + u_c(z_1) \qquad [48]$$

+ - + +

$$\text{s. t.} \sum_i v_{i,1} = 1 \qquad v_{i,1} \geq 0$$

$$\sum_i v_{i,1}\left(a_{1,i}(1+t) + b_i z_1 + \overline{C}\overline{C}m_i\right) = w_1$$

The consumer will obtain more satisfaction as much time he stays on suburb i, as less pollution exists and as more public expense will be realised by local authorities, as well as of the level consumed of y_i. The last equation represents the budgetary restriction of the unskilled workers. They can distribute their incomes in the consumption of housing and the good y_i, payment of taxes and commuting costs to go to work U_1 (.) will be an increasing concave function of $v_{i,1}$, of second class.

2) *Skilled workers:* similarly, the qualified individuals maximise their utility according to the level consumed of v_i (positive relation), consumption of housing in the case of unskilled workers (negative relation), pollution (negative relation) and public expense (positive relation), as well as the consumption of the good y_i. Also U_2(.) will be of second class, concave and increasing in $v_{i,2}$.

$$Max \quad \sum_i u_2\left(v_{i,2}, V_1{*}(v_{i,1}, v_{i,2}), P{*}(P_i, v_{i,2}), G{*}(G_i, v_{i,2})\right) + u_c(z_2) \qquad [49]$$

+ - - + +

$$\text{s. t. } \sum_i v_{i,2} = 1 \qquad v_{i,2} \geq 0$$

$$\sum_i v_{i,2}\left(a_{1,i}(1+t) + b_i z_2 + \overline{C}\overline{C} l_i\right) = w_2$$

3) *Landlords:* the individuals that don't work will maximise their utility in function of the level consumed of v_i by them and by unskilled and skilled workers and of the pollution, as well as the consumption of the good y_i. We suppose as always that U_3(.) is enough smooth. In this case, the public expense doesn't influence their utility, because it is supposed that they have their own infrastructure. So:

$$Max \quad \sum_i u_3\left(\underset{+}{v_{i,3}}, \underset{-}{V_1} * (v_{i,1}, v_{i,3}), \underset{-}{V_2} * (v_{i,2}, v_{i,3}), \underset{-}{P} * (P_i, v_{i,2})\right) + \underset{+}{u_c}(z_3) \qquad [50]$$

$$\text{s. t. } \sum_i v_{i,3} = 1 \qquad v_{i,3} \geq 0$$

$$\sum_i v_{i,3}\left(a_{1,i}(1+t) + b_i z_3 + k'\right) = w_3 = k(1+r) + \frac{an}{I_3}$$

The incomes obtained by these individuals are distributed of similar way at the rest of the individuals, with an exception: one part of that incomes is kept for investment, this is called k'.

If $k' = k$ (that keeps the nominal investment level for an undefined future), then:

$$\sum_i v_{i,3}\left(a_{1,i}(1+t) + b_i z_3\right) = kr + \frac{an}{I_3} \qquad [51]$$

In the other hand, considering the expressions [22] and that the total expense in y is $z_j \sum_i b_i v_{i,j}$, we suppose that each kind of individual expends the same proportion of their incomes in the consumption of the good y_i, so:

$$z_j \sum_i v_{i,j}\left(1 + \frac{a_{1,i}}{\delta}\right) = \Theta_j w_j \qquad [52]$$

and simplifying:

$$z_j\left(1 + \frac{1}{\delta}\sum_i v_{i,j} a_{1,j}\right) = \Theta_j w_j \qquad [53]$$

$$1 + \frac{1}{\delta}\sum_i v_{i,j} a_{1,j} = \frac{\Theta_j w_j}{z_j} \qquad [54]$$

$$\sum_i v_{i,j} a_{1,j} = \left(\frac{\Theta_j w_j - z_j}{z_j} \right) \delta \qquad [55]$$

The expression [55] represents the equilibrium condition or equation of expense. The total expense in housing will be higher as more incomes are obtained by the individuals and as less consumption of the good y_i will be made by them.

2.3. Efficiency loss

There are some commuting cost at this city, that we can analyse as an efficiency loss, because there is not institution or sector who benefices with it.

The Government spend part of his budget (R) improving the quality of public transportation service (train, highways, etc.) at the whole city. If public services are better, the commuting cost will be lower, so

$$CC = f(R) = \overline{C}\,\overline{C} \qquad [56]$$

Is important to clarify the difference between general public services (R) and local ones (G_i): the first are related with all the suburbs, the second ones with some of them. Then, when the government spent more money in G_i the consequences over commuting cost are not clear.

The whole efficiency loss at this city are the commuting cost that the unskilled workers have everyday when they go to work to the factory (distance m_i) and when the skilled ones go to the C.B.D.(distance l_i).

$$PE = \sum_{i=1}^{n} I_1 v_{i,1} m_i \overline{C}\,\overline{C} + \sum_{i=1}^{n} I_2 v_{i,2} l_i \overline{C}\,\overline{C} \qquad [57]$$

$$PE = \overline{C}\,\overline{C} \left(I_1 \sum_{i=1}^{n} v_{i,1} m_i + I_2 \sum_{i=1}^{n} v_{i,2} l_i \right) \qquad [58]$$

As we have said previously everyone buys the y good inside his suburb, so there are not commuting cost in the good purchase.

3. SOME THEORETICAL RESULTS

In this section we advance some of the possible conclusions of the model we presented before. This work is a preliminary one (the model we develop has many different features respect to the standard Muth-Mills literature) and we couldn't get a strong characterisation, but with some additional refining, we hope we can formalise the (partial) equilibrium of this model.

In the following argumentation, we exclude the government forces, because it makes the analysis somewhat complicated, and given the symmetrical distribution we postulate (see 1.6) it doesn't change the resulting asymmetry. Of course a revised version should overcome this working hypothesis.

Our model is very complex as are the forces working there, but to characterise the mechanism of the system, we can say, that for a model of spatial location (Villar, 1996) there are centripetal and centrifugal forces. In contrast with the standard model, now some of them are asymmetrical (the industrial externality) and because there are different kinds of individuals who perceive the different forces with unequal intensity (some of these doesn't affect them all, as the commuting cost to the working places), the solution of the model will not be as simple as the standard model. Apart from the congestion and commuting costs similarly (remember the differences between types) to Muth-Mills', we find al least two important forces: government (centripetal, we have obviated that) and pollution (centripetal but asymmetric respect to the C.B.D.), with some other minor forces (neighbourhood quality).

For convenience we will define the -i suburb as the n+1-i one (that is, the mirror district of i), it is easy to show that $l_i = l_{-i}$, so it is equivalent to talk about suburb symmetry or distance symmetry.

Proposition 1

In equilibrium, agents haven't a symmetrical distribution, that is: $v_{i,j} \neq v_{-i,j}$ for some i, j.

Proof

Suppose we have two suburbs $i \leq n/2$, -i so that

$$v_{i,j} = v_{-i,j} \ \forall\ j \text{ so } v_{i,3} = v_{-i,3} \qquad [60]$$

Obviously al least one suburb must have a strict positive residence time for the landlord type, let us say i, and of course if the symmetrical area -i has a 0 density we have proved our assertion, so as the density must be the same. As the price is a monotonous decreasing function of h_i:

$$\sum_j v_{i,j} = \sum_j v_{-i,j} \quad \Rightarrow \quad h_i = h_{-i} \quad \Rightarrow \quad a_{1,i} = a_{1,-i} \qquad [61]$$

For j=3, by [22] we have,

$$b_j = 1 + \frac{a_{1,i}}{\delta} \Rightarrow \sum_i v_i \left(a_{1,i} + \left(1 + \frac{a_{1,i}}{\delta} \right) z_3 \right) = z_3 + a_{1,i} \left(1 + \frac{z_3}{\delta} \right) \qquad [62]$$

The first order conditions evaluated in our selected optima ($v_{i,j} = v_{-i,j}$, $\forall$j) are:

$$\frac{\frac{\partial u_3}{\partial v_{i,3}}(v_{i,3}, V_1^*(v_{i,1}, v_{i,3}), V_2^*(v_{i,2}, v_{i,3}), P^*(P_i, v_{i,3})) + \frac{\partial u_3}{\partial V_1^*}(.)\frac{\partial V_1^*}{\partial v_{i,3}}(.) + \frac{\partial u_3}{\partial V_2^*}(.)\frac{\partial V_2^*}{\partial v_{i,3}}(.) + \frac{\partial u_3}{\partial P^*}(.)\frac{\partial P^*}{\partial v_{i,3}}(.)}{\frac{\partial u_3}{\partial v_{-i,3}}(.) + \frac{\partial u_3}{\partial V_1^*}(.)\frac{\partial V_1^*}{\partial v_{-i,3}}(.) + \frac{\partial u_3}{\partial V_2^*}(.)\frac{\partial V_2^*}{\partial v_{-i,3}}(.) + \frac{\partial u_3}{\partial P^*}(.)\frac{\partial P^*}{\partial v_{-i,3}}(.)} = \frac{z_3 + a_{1,i}\left(1 + \frac{z_3}{\delta}\right)}{z_3 + a_{1,-i}\left(1 + \frac{z_3}{\delta}\right)} \qquad [63]$$

As $a_{1,i} = a_{1,-i}$ the right hand side of the equation must be 1, so the denominator of the left side must be equal to the numerator. As we are evaluating the same functions in the same points (the instrumental functions we used are not related to the neighbourhood) the first three members of both sides are the same, so we get:

$$\frac{\partial u_3}{\partial P^*}(.)\ \frac{\partial P^*}{\partial v_{i,3}}(P_i, v_{i,3}) = \frac{\partial u_3}{\partial P^*}(.)\ \frac{\partial P^*}{\partial v_{-i,3}}(P_{-i}, v_{-i,3}) \qquad [64]$$

Which can’t be true, because we are evaluating the derivative of the same function P* in two different points (we can say even more, the l.h.s. must be greater than the r.h.s., because, by hypothesis, the last is 0 for all $v_{i,3}$ so its derivative must be 0, and as the derivative in the second element in the l.h.s. is a increasing function, it must be positive, the utility is decreasing in the pollution, so the l.h.s. must be negative)

This proposition although simple and not very limitative for the equilibrium, is a interesting one; because we suppose there are asymmetric forces in the model, the individuals' behaviour is asymmetric, if they confront with the same prices they will prefer the least polluted area. Observe that the condition for an interior solution isn’t absolutely necessary, the real difference is the effect of the contamination. This simple condition says us that the standard Muth-Mills model is not valid in this context. But of course we are not saying too much for the equilibrium prices.

Proposition 2

The city is not symmetrical (with respect to the city centre), so ∃ i ≤ n/2 , -i such as $a_{1,i} \neq a_{1,-i}$.

Proof:

For simplicity, we suppose that the solution for the problem is interior for at least these two neighbourhoods, although the argumentation is very similar in other case.

If the city is symmetrical $a_{1,i} = a_{1,-i}$ so we have for j=3, as we have proved in proposition 1,

$$\begin{aligned}&\frac{\partial u_3}{\partial v_{i,3}}(v_{i,3}, V_1^*(v_{i,1}, v_{i,3}), V_2^*(v_{i,2}, v_{i,3}), P^*(P_i, v_{i,3})) + \frac{\partial u_3}{\partial V_1^*}(.)\frac{\partial V_1^*}{\partial v_{i,3}}(.) + \frac{\partial u_3}{\partial V_2^*}(.)\frac{\partial V_2^*}{\partial v_{i,3}}(.) + \frac{\partial u_3}{\partial P^*}(.)\frac{\partial P^*}{\partial v_{i,3}}(.)\\ &= \frac{\partial u_3}{\partial v_{-i,3}}(.) + \frac{\partial u_3}{\partial V_1^*}(.)\frac{\partial V_1^*}{\partial v_{-i,3}}(.) + \frac{\partial u_3}{\partial V_2^*}(.)\frac{\partial V_2^*}{\partial v_{-i,3}}(.) + \frac{\partial u_3}{\partial P^*}(.)\frac{\partial P^*}{\partial v_{-i,3}}(.)\end{aligned} \qquad [65]$$

As we have shown before, if $v_{i,3} = v_{-i,3}$ the r.h.s. is bigger than the l.h.s., so if the equality holds as u_3 is a concave increasing function in $v_{i,3}$, we get that $v_{i,3}$ must be inferior to $v_{-i,3}$.

For j=2, the argumentation is very similar,

$$\frac{\frac{\partial u_2}{\partial v_{i,2}}(v_{i,2}, V_1^*(v_{i,1}, v_{i,2}), P^*(P_i, v_{i,2})) + \frac{\partial u_2}{\partial V_1^*}(.)\frac{\partial V_1^*}{\partial v_{i,2}}(.) + \frac{\partial u_2}{\partial P^*}(.)\frac{\partial P^*}{\partial v_{i,2}}(.)}{\frac{\partial u_2}{\partial v_{-i,2}}(.) + \frac{\partial u_2}{\partial V_1^*}(.)\frac{\partial V_1^*}{\partial v_{-i,2}}(.) + \frac{\partial u_2}{\partial P^*}(.)\frac{\partial P^*}{\partial v_{-i,2}}(.)} = \frac{z_2 + a_{1,i}\left(1 + \frac{z_2}{\delta}\right) + \overline{C}\overline{C}l_i}{z_2 + a_{1,-i}\left(1 + \frac{z_2}{\delta}\right) + \overline{C}\overline{C}l_{-i}} \qquad [66]$$

As $l_i = l_{-i}$ the r.h.s. is equal to 1, so we have

$$\frac{\partial u_2}{\partial v_{i,2}}(.)+\frac{\partial u_2}{\partial V_1^*}(.)\frac{\partial V_1^*}{\partial v_{i,2}}(.)+\frac{\partial u_2}{\partial P^*}(.)\frac{\partial P^*}{\partial v_{i,2}}(.)=\frac{\partial u_2}{\partial v_{-i,2}}(.)+\frac{\partial u_2}{\partial V_1^*}(.)\frac{\partial V_1^*}{\partial v_{-i,2}}(.)+\frac{\partial u_2}{\partial P^*}(.)\frac{\partial P^*}{\partial v_{-i,2}}(.) \qquad [67]$$

Now if $v_{i,2} = v_{-i,2}$ the equation cannot hold, so $v_{i,2}$ must be inferior to $v_{-i,2}$.

At last, for the unskilled workers (j=1) we have,

$$\frac{\frac{\partial u_1}{\partial v_{i,1}}(v_{i,1},P^*(P_i,v_{i,1}))+\frac{\partial u_1}{\partial P^*}(.)\frac{\partial P^*}{\partial v_{i,1}}(.)}{\frac{\partial u_1}{\partial v_{-i,1}}(.)+\frac{\partial u_1}{\partial P^*}(.)\frac{\partial P^*}{\partial v_{-i,1}}(.)}=\frac{z_1+a_{1,i}\left(1+\frac{z_1}{\delta}\right)+\overline{C}\,\overline{C}m_i}{z_1+a_{1,-i}\left(1+\frac{z_1}{\delta}\right)+\overline{C}\,\overline{C}m_{-i}} \qquad [68]$$

Notice that the r.h.s. is inferior to one as m_i is inferior to m_{-i} (we suppose i as a south area), we get

$$\frac{\partial u_1}{\partial v_{i,1}}(.)+\frac{\partial u_1}{\partial P^*}(.)\frac{\partial P^*}{\partial v_{i,1}}(.)<\frac{\partial u_1}{\partial v_{-i,1}}(.)+\frac{\partial u_1}{\partial P^*}(.)\frac{\partial P^*}{\partial v_{-i,1}}(.) \qquad [69]$$

In general we can't prove without some additional restrictions on the functions of cost and utility, that $v_{i,1}$ must be inferior to $v_{-i,1}$, because the forces of pollution and commuting cost are opposed, but al least we suppose that for some i to get [68] we need $v_{i,1} = v_{-i,1}$.

If this is credible, we have proved that if $a_{1,i} = a_{1,-i}$, $h_i = \sum_j I_i v_{i,j} < h_{-i} = \sum_j I_i v_{-i,j}$ [70], which is a contradiction, as $F_i<F_{-i}$ ($z_{i,j}$ is not decreasing in $v_{i,j}$). Observe that the last supposition isn't necessary as we only need to find a suburb with different prices that it's mirror, so if for some i the forces exactly oppose each one to get [70] with equality, for the next area i+1 or i-1 the equality can' t hold so we have proved our affirmation

With these two propositions we have shown that a city as described will not be symmetrical (remember we are talking exclusively on a static equilibria and from a partial perspective, as we have never try to close this model), but we shouldn't have characterised the equilibrium prices (the Muth-Mills model proved that they are a decreasing function of the distance to the city centre), this part of the job is pending, but we could speculate:

Proposition 3

In equilibria, the n/2 first neighbourhoods prices are increasing in i, so $a_{1,i} > a_{1,j}$ $\forall$ $n/2 \geq i > j \geq 1$. This is equivalent to $h_1 \geq h_2 \geq \ldots \geq h_{n/2}$.

The main point of this proposition is the need to prove that the relative forces acting over the skilled workers let them get a inferior congestion on the centre, paying a superior price for that.

We are not proposing exactly a inverse Muth-Mills model as we probably can find some areas with an density inferior to the centre (in the North), but in the South there is a high density of unskilled workers in the edge of town, with some pollution problems, and paying a minimum commuting cost.

A good test for this model should be a simulation on a concrete city with more definition of its functions, but we have preferred to relegate that to further investigation. Another line of work must be a dynamic model or at least a short term simulation.

4. FINAL REMARKS

Although we know this a quite preliminary paper, and there are many rough edges, we try to open a new line of research about the causes of the growth of the cities outside of the standard focus. Perhaps this paper looks so simple and noisy, we think there are many possibilities for future studies under dynamic modelization and further simulation.

5. REFERENCES

FUJITA, M. y THISSE, J.F. (1996): Economics of agglomeration, *Journal of the Japanese an International Economies*, 10, pp. 339-378.

MILLS, E.S. (1967): An aggregate model of resource allocation in a metropolitan area, *American Economic Review*, nº 57, pp. 197-210.

MUTH, R.F. (1969): Cities and Housing, *Chicago University Press*, Chicago.

TIEBOUT, C. (1956): A pure theory of local expenditures, *Journal of Political Economy*, nº 64, pp. 416-424.

TURNBULL, G.K. (1990): The pure theory of household, location: an axiomatic approach, *Journal of Regional Science*, vol. 30, nº 4, pp. 549-562.

VILLAR, O. (1996): Configuration of cities: the effects of congestion cost and government, *Working papers 96-17*, Universidad Carlos III, Madrid.

WILKINSON, R.K. & ARCHER, C.A. (1973): Measuring the determinants of relative houses prices, *Environment and Planning*, nº 5, pp. 357-367.

SERIES DE DOCUMENTOS DE TRABAJO PUBLICADOS

WP 9801/Nº 1

PROPUESTA DE UN ANÁLISIS ECONOMÉTRICO PARA EL ESTUDIO DEL PRECIO DE LA VIVIENDA URBANA

Pablo Brañas Garza; Pablo Fernández-Álvarez; José Mª Caridad y Ocerin

WP 9802/Nº 2

UN ANÁLISIS DEL CRECIMIENTO Y LA CONVERGENCIA DE LA ECONOMÍA ANDALUZA ENTRE 1985 Y 1995

José García Roa

WP 9803/Nº 3

PHYSICAL AND NOT SO PHYSICAL DISTANCES IN A SIMPLE URBAN MODEL: AN ANALYSIS

Pablo Brañas Garza; Javier Rodero Cosano; Joan Carles Martori

WP 9804/Nº 4

UNA EVALUACIÓN DEL CAMBIO DE ESTRATEGIA DE LA POLÍTICA MONETARIA EN ESPAÑA: PERSPECTIVAS DE FUTURO

Antonio Martín Mesa; Francisco Alcalá Olid

WP 9805/Nº 5

URBAN MICROECONOMICS WITHOUT MUTH-MILLS: A NEW THEORETICAL FRAME (OR THE GREAT WALKOVER)

Javier Rodero Cosano; Pablo Brañas Garza; Inmaculada Fernández Piñar

WP 9806/Nº 6

LAS EXTERNALIDADES URBANAS: ENTRE ALPEROVICH Y FUJITA

Pablo Brañas Garza; Alejandro Lorca Corrons; Javier Rodero Cosano; Mª Angustias Dávila Vargas-Machuca

WP 9807/Nº 7

LA ECONOMIA ISLÁMICA Y SUS CONTRATOS: UNA PANORÁMICA

Pablo Brañas Garza; Alejandro Lorca Corrons; Javier Rodero Cosano

WP 9808/Nº8

SIZE, PROFITABILITY AND AGENCY PROBLEMS IN PROFIT LOSS SHARING IN ISLAMIC

Humayon A. D; David I. Harvey; John R. Presley

WP 9901/Nº9

CAPITAL HUMANO Y CRECIMIENTO EN EL MEDITERRÁNEO: ¿*SPILLOVERS* O DETERMINISMO GEOGRÁFICO

Javier Rodero Cosano, Pablo Brañas Garza, Mª Lucia Cabañes Argudo, Alejandro V. Lorca Corrons

WP 9902/Nº10

SOBRE EL RUIDO Y SU PERCEPCIÓN: UNA APROXIMACIÓN EXPERIMENTAL

Pablo Brañas Garza; M. D. Alcántara Moral y Javier Rodero Cosano

WP 9903/Nº11

CRECIMIENTO ECONÓMICO ENDÓGENO Y CAPITAL PÚBLICO DESDE UNA PERSPECTIVA REGIONAL: UNA APROXIMACIÓN

Diego Martínez López

WP 0001/Nº12

DIFFERENT PATHS OF URBAN AGGLOMERATION IN SPANISH REGIONS: EVIDENCE FROM 1960-1998

Pablo Brañas Garza y Francisco Alcalá Olid

WP 0002/Nº13

IS THERE ANY RELATIONSHIP BETWEEN PUBLIC INVESTMENT AND ECONOMIC GROWTH IN THE SPANISH REGIONS?

Diego Martínez López

WP 0003/Nº14

CONTRACTS IN THE AGRICULTULTURAL SECTOR WITH MORAL HAZARD AND HIDDEN INFORMATION: SPECULATIONS, TRUTHS AND RISK-SHARING.

Francisca Jiménez Jiménez

WP 0004/Nº15

HOTELLING AND THE OLYMPUS: MODELLING DIFFERENCES IN RELIGIOUS PRICE

Javier Rodero y Pablo Brañas Garza

WP 0005/Nº16

AN EMPIRICAL MEASUREMENT OF THE EFFECTS OF EXTERNALITIES ON LOCATION CHOICE

Pablo Brañas Garza y Javier Rodero

WP 0006/Nº17

EL ENDEUDAMIENTO A LARGO PLAZO DE LA HACIENDA PÚBLICA ANDALUZA: UNA VISIÓN PANORÁMICA

Diego Martínez López