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The uncertain probabilistic OWA distance operator and its application in group decision making

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ABSTRACT

In this paper, we present the uncertain probabilistic ordered weighted averaging distance (UPOWAD) operator. Its main advantage is that it uses distance measures in a unified framework between the probability and the OWA operator that considers the degree of importance of each concept in the aggregation. Moreover, it is able to deal with uncertain environments represented in the form of interval numbers. We study some of its main properties and particular cases such as the uncertain probabilistic distance (UPD) and the uncertain OWA distance (UOWAD) operator. We end the paper by presenting an application to a group decision making problem regarding the selection of robots.

1. Introduction

The distance measures are a common tool for measuring the deviations of different arguments in decision making. The main advantage of using distance measures in decision making is that we can compare the alternatives of the problem with some ideal results [1]. Through this comparison, the alternative with the closest result to the ideal is the optimal choice. Over the past several decades, a variety of distance measures have been introduced and investigated [2–12], among them, the Hamming distance measure [13] is one of the most popular distance measures. Usually, when using the distance measure in decision making, we normalize it by using the arithmetic mean or the weighted average (WA) obtaining the normalized Hamming distance (NHD) and the weighted Hamming distance (WHD), respectively. However, it is sometimes of interest to consider the possibility of parameterizing the results from the maximum distance to the minimum distance. A very useful technique that can provide a parameterized family of aggregation operators that includes the maximum, the minimum, the average and others, is the ordered weighted averaging (OWA) operator [14]. Recently, on the basis of the idea of the OWA operator, Merigó and Gil-Lafuente [15] introduced an ordered weighted averaging distance (OWAD) operator, and applied it to decision making problem about selecting financial products. The main advantage of this approach is that we are able to underestimate or overestimate the selection process according to the desired degree of optimism (i.e., the degree of orness). Another main advantage of the OWAD is that it can provide a parameterized family of aggregation operators ranging from the minimum to the maximum. Thus, the decision maker is able to consider the decision problem more clearly according to his or her interests in the aggregation process. Since it was introduced, the OWAD has been receiving increasing attention. For example, Merigó and Casanovas [16] extended this approach by using linguistic variables, and developed the linguistic ordered weighted averaging distance (LOWAD) operator. They also developed a generalization by using induced aggregation

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operators [17]. Furthermore, they also extended this approach by using the Euclidean distance [6] and the Minkowski distance [18]. Zeng and Su [19] considered the situation with intuitionistic fuzzy information, and developed an intuitionistic fuzzy ordered weighted distance (IFOWD) operator. Merigó and Gil-Lafuente developed an application of the OWAD in sport management [20] and in human resource management [21]. Yager [22] generalized it by using norms. Xu and Xia analyzed the use of hesitant fuzzy sets in the OWAD operator [23].

Recently, Merigó [24] suggested a new model called the probabilistic OWA (POWA) operator, which unifies the OWA operator and the probability in the same formulation considering the degree of importance that each concept has in the aggregation. Thus, we can use the attitudinal character of the decision maker and the probabilistic information of the specific problem considered. In the context of the POWA operator, it is assumed that information is exactly known and can be represented with exact numbers or singletons. However, this assumption may not characterize real-world situations. Therefore, it is necessary to use another approach to represent situations with high degrees of uncertainty. In Ref. [25], Merigó and Wei suggested a method for representing uncertainty in the POWA operator using interval numbers, and developed the uncertain probabilistic OWA (UPOWA) operator. The main advantage of the UPOWA operator is that it provides more complete information to the decision maker by using interval numbers that includes a wide range of results and by using probabilities and OWA operators in the same formulation considering the degree of importance of each concept in the aggregation. Thus, we are able to consider objective information (probabilistic) and the attitudinal character of the decision maker in the same formulation. Merigó [26] also considered the fuzzy situations and developed a fuzzy probabilistic OWA (FPOWA) operator. Furthermore, he developed an uncertain probabilistic weighted average (UPWA) operator and applied it to multi-person decision making problem regarding the selection of strategies by using the theory of expertons [27].

The aim of this paper is to develop the uncertain probabilistic OWA distance (UPOWAD) operator. It is a new extension of the UPOWA operator by using distance measures in the analysis. Therefore, it includes uncertain information assessed with interval numbers, the probability and the OWA operator at the same time in the Hamming distance. Thus, a more complete formulation of the Hamming distance is obtained because it can consider a parameterized family of operators between the maximum and the minimum and the degree of importance that each argument has in the analysis. Moreover, it also permits to analyze the distance measures in a probabilistic way. This can be useful in a lot of situations, especially when the information can be assessed in an objective way. Note that in this paper we consider the use of the Hamming distance but it is also possible to consider other distance measures such as the Euclidean and the Minkowski distance. The main advantage of using distance measures is that we can compare the real-world information with ideal information and see which alternative better fits with the interests of the decision-maker. For example, in human resource selection, we can establish an ideal candidate that would perfectly fit the company and compare it with the real-world alternatives that we have in the market and select the candidate with closest results to the ideal one.

We study the applicability of the new model and we see that it is very broad because all the studies that use the probability, the OWA operator or distance measures can be revised and extended by using this new framework. We also present an application of the new approach to a group decision making problem concerning selection of robots. The main advantage of this model is that it gives a more complete view of the decision problem because it considers a wide range of distance aggregation operators according to the interests of the decision maker. Moreover, by using several experts in the analysis, we obtain information that it is more robust because the opinion of several experts is always better than the opinion of one. We see that depending on the particular type of UPOWAD operator used, the results may be different leading to different decisions.

This paper is organized as follows. In Section 2, we briefly review some basic concepts about the Hamming distance, interval numbers, the UPOWA and the OWAD operator. In Section 3 we introduce the UPOWAD operator, and different families of UPOWAD operators are analyzed in Section 4. In Section 5 we develop an application in a decision making problem and present a numerical example. Section 6 summarizes the main conclusions of the paper.

2. Preliminaries

In this Section we briefly review the Hamming distance, interval numbers, the UPOWA and the OWAD operator.

2.1. The Hamming distance

The Hamming distance [13] is a useful technique for calculating the differences between two parameters, such as problems with two elements or two sets. For two sets $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{b_1, b_2, \ldots, b_n\}$, the weighted Hamming distance (WHD) can be defined as follows:

Definition 1. A weighted Hamming distance measure of dimension n is a mapping WHD: $R^n \times R^n \to R$ that has an associated weighting vector W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$d_{\mathrm{WHD}}(A,B) = \sum_{i=1}^{n} w_i |a_i - b_i|, \tag{1}$$

where a_i and b_i are the ith arguments of the sets A and B, respectively. Note that if $w_j = 1/n$, the we get the normalized Hamming distance (NHD).

2.2. The OWA operator

The OWA operator [14] provides a parameterized family of aggregation operators that include the maximum, the minimum and the average criteria as special cases. Since its appearance, the OWA operator has been used in a wide range of applications [6,9,15–24,26–48]. It can be defined as follows:

Definition 2. An OWA operator of dimension n is a mapping OWA: $R^n \to R$ that has an associated weighting W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$\mathrm{OWA}(a_1, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{2}$$

where b_j is the jth largest of the a_i.

The OWA operator aggregates the information according to the attitudinal character (or degree of orness) of the decision maker [14]. The attitudinal character is represented according to the following formula:

$$\alpha(W) = \sum_{j=1}^{n} w_j \left(\frac{n-j}{n-1} \right). \tag{3}$$

Note that $\alpha(W) \in [0,1]$. The more weight W is located close to the top, the closer α is to 1. In decision-making problems, the degree of orness is useful for representing the attitudinal character of the decision-maker by using it as the degree of optimism or pessimism.

2.3. The ordered weighted averaging distance (OWAD) operator

The OWAD (or Hamming OWAD) operator [15] is an extension of the traditional normalized Hamming distance by using OWA operators. The main difference is the reordering of the arguments of the individual distances according to their values. For two sets $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{b_1, b_2, \ldots, b_n\}$, the OWAD operator can be defined as follows:

Definition 3. An OWAD operator of dimension n is a mapping OWAD: $R^n \times R^n \to R$ that has an associated weighting vector W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$\mathrm{OWAD}(\langle a_1, b_1 \rangle, \langle a_2, b_2 \rangle, \ldots, \langle a_n, b_n \rangle) = \sum_{j=1}^{n} w_j d_j, \tag{4}$$

where d_j is the jth largest of the $|a_i - b_i|$.

2.4. The interval number

The interval number [49,50] is a very useful and simple technique for representing the uncertainty. By using interval numbers we can consider a wide range of possible results between the maximum and the minimum. Note that in the literature, there are a lot of studies dealing with uncertain information represented in the form of interval numbers [9,25,27,34,38,39,43,51–56]. The interval number can be defined as follows.

Definition 4. Let $\tilde{a} = [a^L, a^U] = \{x | 0 \leqslant a^L \leqslant a^U\}$, then $\tilde{a}$ is called an interval number. Especially, $\tilde{a}$ is a nonnegative real number, if $a^L = a^U$.

In the following, we are going to review some basic interval number operations as follows: Let $\tilde{a} = [a^L, a^U]$ and $\tilde{b} = [b^L, b^U]$ be interval numbers, then

(1) $\tilde{a} + \tilde{b} = [a^L + b^L, a^U + b^U]$;
(2) $\lambda \tilde{a} = [\lambda a^L, \lambda a^U]$,

where $\lambda \geqslant 0$.

In order to measure the distance between interval numbers, Xu [43] introduced a measure involving each pair of interval numbers as following:

Definition 5. Let $\tilde{a} = [a^L, a^U]$ and $\tilde{b} = [b^L, b^U]$ be two interval numbers, then

$$d_{UD}(\tilde{a}, \tilde{b}) = \frac{1}{2} \left(|a^L - b^L| + |a^U - b^U| \right) \tag{5}$$

is called the uncertain distance between $\tilde{a}$ and $\tilde{b}$.

Additionally, Merigó and Wei [25] and Merigó and Casanovas [34] establish the following criterion for reordering interval numbers: if $\frac{a^L+a^U}{2} > \frac{b^L+b^U}{2}$, then $\tilde{a} > \tilde{b}$; In the case of a tie, we select the interval with the lowest difference, i.e., if $(a^U - a^L) > (b^U - b^L)$, then $\tilde{a} > \tilde{b}$.

Let Ω be the set of all interval numbers, $\tilde{A} = (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)$ and $\tilde{B} = (\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_n)$ be two sets of interval numbers. Then we can define an uncertain weighted distance (UWD) between $\tilde{A}$ and $\tilde{B}$ as following:

Definition 6. An UWD of dimension n is a mapping UWD: $\Omega^n \times \Omega^n \to R$ that has an associated weighting vector W with $w_j \in [0,1]$ and $\sum_{j=1}^n w_j = 1$, such that

$$d_{\text{UWD}}(\langle \tilde{a}_1, \tilde{b}_1 \rangle, \langle \tilde{a}_2, \tilde{b}_2 \rangle, \ldots, \langle \tilde{a}_n, \tilde{b}_n \rangle) = \sum_{i=1}^n w_i d_{UD}(\tilde{a}_i, \tilde{b}_i), \tag{6}$$

where $\tilde{a}_i$ and $\tilde{b}_i$ are the ith arguments of the sets $\tilde{A}$ and $\tilde{B}$, respectively. $d_{\text{UD}}(\tilde{a}_i, \tilde{b}_i)$ is the distance between $\tilde{a}_i$ and $\tilde{b}_i$. Specially, if $w_j = 1/n$, the we get the uncertain normalized distance (UND) between $\tilde{A}$ and $\tilde{B}$.

2.5. The uncertain probabilistic OWA operator

The uncertain probabilistic ordered weighted averaging (UPOWA) operator [25] is an aggregation operator that uses uncertain information in the aggregation process by using interval numbers in the POWA operator. Therefore, it is an extension of the OWA operator for situations where we find probabilistic and uncertain information that can be assessed with interval numbers. Its main advantage is that it can unify both concepts considering the degree of importance that they have in the specific problem considered. It can be defined as follows:

Definition 7. An UPOWA operator of dimension n is a mapping UPOWA: $\Omega^n \to \Omega$ that has an associated weighting vector W with $w_j \in [0,1]$ and $\sum_{j=1}^n w_j = 1$, according to the following formula:

$$\text{UPOWA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \sum_{j=1}^n \hat{p}_j \tilde{b}_j, \tag{7}$$

where $\tilde{b}_j$ is the jth largest of the $\tilde{a}_i$, the $\tilde{a}_i$ are interval numbers and each one has an associated probability p_i with $\sum_{i=1}^n p_i = 1$ and $p_i \in [0,1]$, $\hat{p}_j = \beta w_j + (1-\beta)p_j$ with $\beta \in [0,1]$ and p_j is the probability p_i ordered according to $\tilde{b}_j$, that is, according to the jth largest of the $\tilde{a}_i$.

By choosing a different manifestation in the weighting vector, we are able to obtain a wide range of particular types of UPOWA operators [25]. Especially, when $\beta = 0$, we get the uncertain probabilistic aggregation, and if $\beta = 1$, we get the uncertain OWA (UOWA) operator [44].

3. The uncertain probabilistic OWA distance (UPOWAD) operator

The uncertain probabilistic ordered weighted averaging distance (UPOWAD) operator is a distance measure that uses a unified framework between the probability and the OWA operator in the normalization process of the Hamming distance. Thus, we can use probabilistic information, the attitudinal character of the decision maker and distance measures in the same formulation. Moreover, it also uses uncertain information assessed with interval numbers in the aggregation process. Therefore, it can assess complex environments where the information is very imprecise and cannot be assessed with exact numbers but it is possible to use interval numbers. It includes a wide range of particular cases such as the uncertain probabilistic maximum distance, the uncertain probabilistic minimum distance, the uncertain probabilistic aggregation distance and the UOWAD operator. Therefore, we can get a more general formulation of the uncertain distance that considers a lot of possible situations depending on the interests of the decision makers. Let $\tilde{A} = (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)$ and $\tilde{B} = (\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_n)$ be two sets of interval numbers, the UPOWAD operator can be defined as follows.

Definition 8. An UPOWAD operator of dimension n is a mapping UPOWAD: $\Omega^n \times \Omega^n \to R$ that has an associated weighting vector W with $w_j \in [0,1]$ and $\sum_{j=1}^n w_j = 1$, according to the following formula:

$$\text{UPOWAD}(\langle \tilde{a}_1, \tilde{b}_1 \rangle, \langle \tilde{a}_2, \tilde{b}_2 \rangle, \ldots, \langle \tilde{a}_n, \tilde{b}_n \rangle) = \sum_{j=1}^n \hat{p}_j d_{UD}(\tilde{a}_j, \tilde{b}_j), \tag{8}$$

where $d_{UD}(\tilde{a}_j, \tilde{b}_j)$ is the jth largest of the $d_{UD}(\tilde{a}_i, \tilde{b}_i)$, the $d_{UD}(\tilde{a}_i, \tilde{b}_i)$ are the distances between $\tilde{a}_i$ and $\tilde{b}_i$, and each one has an associated probability p_i with $\sum_{i=1}^n p_i = 1$ and $p_i \in [0,1]$, $\hat{p}_j = \beta w_j + (1-\beta)p_j$ with $\beta \in [0,1]$ and p_j is the probability p_i ordered according to $d_{UD}(\tilde{a}_j, \tilde{b}_j)$, that is, according to the jth largest of the $d_{UD}(\tilde{a}_i, \tilde{b}_i)$.

Note that it is also possible to formulate the UPOWAD operator separating the part that strictly affects the OWA operator and the part that affects the probabilities. This representation is useful to see both models in the same formulation but it does not seem to be as a unique equation that unifies both models.

Definition 9. An UPOWAD operator is a mapping UPOWAD: $\Omega^n \times \Omega^n \rightarrow R$ of dimension n, if it has an associated weighting vector W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$ and a probabilistic vector V, with $\sum_{i=1}^{p} p_i = 1$ and $p_i \in [0,1]$, such that:

$$\mathrm{UPOWAD}(\langle\tilde{a}_1,\tilde{b}_1\rangle,\langle\tilde{a}_2,\tilde{b}_2\rangle,\ldots,\langle\tilde{a}_n,\tilde{b}_n\rangle) = \beta\sum_{j=1}^{n} w_j d_{UD}(\tilde{a}_j,\tilde{b}_j) + (1-\beta)\sum_{i=1}^{n} p_i d_{UD}(\tilde{a}_i,\tilde{b}_i), \tag{9}$$

where $d_{UD}(\tilde{a}_j,\tilde{b}_j)$ is the jth largest of the $d_{UD}(\tilde{a}_i,\tilde{b}_i)$, the $d_{UD}(\tilde{a}_i,\tilde{b}_i)$ are the distances between $\tilde{a}_i$ and $\tilde{b}_i$, and $\beta \in [0,1]$.

It is worth noting that in the literature there are other methods that unify the OWA operator with the probability such as the immediate probability [32,57,58]. Following this approach, we can also extend it by using interval numbers and distance measures forming the uncertain immediate probabilistic distance (UIPD). It can be defined as follows.

Definition 10. An UIPD operator of dimension n is a mapping UIPD: $\Omega^n \times \Omega^n \rightarrow R$ that has an associated weighting vector W of dimension n with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$\mathrm{UIPD}(\langle\tilde{a}_1,\tilde{b}_1\rangle,\langle\tilde{a}_2,\tilde{b}_2\rangle,\ldots,\langle\tilde{a}_n,\tilde{b}_n\rangle) = \sum_{j=1}^{n} \hat{p}_j d_{UD}(\tilde{a}_j,\tilde{b}_j), \tag{10}$$

where $d_{UD}(\tilde{a}_j,\tilde{b}_j)$ is the j th largest of the $d_{UD}(\tilde{a}_i,\tilde{b}_i)$, each $d_{UD}(\tilde{a}_i,\tilde{b}_i)$ has associated a probability p_i, p_j is the associated probability of $d_{UD}(\tilde{a}_j,\tilde{b}_j)$, and $\hat{p}_j = (w_j p_j / \sum_{j=1}^{n} w_j p_j)$.

The main advantage of the UPOWAD operator against the UIPD is that it permits to unify both concepts considering the degree of importance that each of them has in the analysis. On the other hand, the UIPD unifies both concepts but it is more rigid because it doesn't allow different degrees of importance between the OWA and the probability.

Furthermore, we could also consider other methods that unify the OWA operator with the weighted average since the probabilistic aggregation follows a similar methodology than the weighted average. For example, we could consider the hybrid averaging (HA) operator [44] and the WOWA operator [59]. With the HA operator we get the uncertain hybrid weighted distance (UHWD) operator already introduced by Xu [9] and the uncertain weighted OWA distance (UWOWAD) operator. When comparing these methods with the UPOWAD operator, again the main advantage of the UPOWAD is its flexibility by allowing different degrees of relevance between the OWA and the probability.

Note that if the weights of the probabilities and the OWA are also uncertain, then, we have to establish a method for dealing with these uncertain weights. Note that in these situations it is very common that $W = \sum_{j=1}^{n} \tilde{w}_j \neq 1$ and $P = \sum_{j=1}^{n} \tilde{p}_j \neq 1$. Thus, a very useful method for dealing with these situations is by using:

$$\mathrm{UPOWAD}(\langle\tilde{a}_1,\tilde{b}_1\rangle,\langle\tilde{a}_2,\tilde{b}_2\rangle,\ldots,\langle\tilde{a}_n,\tilde{b}_n\rangle) = \frac{\beta}{W}\sum_{j=1}^{n} w_j d_{UD}(\tilde{a}_j,\tilde{b}_j) + \frac{(1-\beta)}{P}\sum_{i=1}^{n} p_i d_{UD}(\tilde{a}_i,\tilde{b}_i). \tag{11}$$

In the following example, we present a simple numerical example showing how to use the UPOWAD operator in an aggregation process. We consider the aggregation with both definitions.

Example 1. Assume the following arguments in an aggregation process: $\tilde{A} = (\tilde{a}_1,\tilde{a}_2,\tilde{a}_3,\tilde{a}_4) = ([0.1,0.5], [0.2,0.6], [0.1,0.7], [0.4,0.5])$, $\tilde{B} = (\tilde{b}_1,\tilde{b}_2,\tilde{b}_3,\tilde{b}_4) = ([0.2,0.8], [0.2,0.7], [0.4,0.5], [0.3,0.7])$. Assume the following weighting vector $W = (0.2,0.2,0.3,0.3)$ and the following probabilistic weighting vector $P = (0.2,0.4,0.1,0.3)$. Note that the probabilistic information has a degree of importance of 70% while the weighting vector W a degree of 30%. If we calculate the distance between $\tilde{A}$ and $\tilde{B}$ by using the UPOWAD operator, we will get the following. The aggregation can be solved either with the Eq. (8), (9). First we should calculate the distances for each pair of interval numbers:

$$d_{UD}(\tilde{a}_1,\tilde{b}_1) = \frac{1}{2}(|0.1-0.2| + |0.5-0.8|) = 0.2.$$

Similarly, we have

$$d_{UD}(\tilde{a}_2,\tilde{b}_2) = 0.05, d_{UD}(\tilde{a}_3,\tilde{b}_3) = 0.25, d_{UD}(\tilde{a}_4,\tilde{b}_4) = 0.15.$$

With Eq. (8) we calculate the new weighting vector as:

$$\hat{v}_1 = 0.3\times0.2 + 0.7\times0.1 = 0.13, \hat{v}_2 = 0.3\times0.2 + 0.7\times0.2 = 0.2,$$

$$\hat{v}_3 = 0.3\times0.3 + 0.7\times0.3 = 0.3, \hat{v}_4 = 0.3\times0.3 + 0.7\times0.4 = 0.37.$$

And then, we calculate the aggregation process as follows:

$$\mathrm{UPOWAD}\left(\tilde{A},\tilde{B}\right) = 0.13\times0.25 + 0.2\times0.2 + 0.3\times0.15 + 0.37\times0.05 = 0.136.$$

With Eq. (9), we aggregate as follows:

$$\mathrm{UPOWAD}(\tilde{A},\tilde{B}) = 0.3\times(0.2\times0.25+0.2\times0.2+0.3\times0.15+0.3\times0.05) + 0.7\times(0.2\times0.2+0.4\times0.05+0.1\times0.25+0.3\times0.15) = 0.136.$$

Obviously, we get the same results with both methods.

From a generalized perspective of the reordering step, we can distinguish between the descending UPOWAD (DUPOWAD) operator and the ascending UPOWAD (AUPOWAD) operator by using $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the DUPOWAD and w^*_{n-j+1} the jth weight of the AUPOWAD operator. Similar to the OWAD operator, the UPOWAD operator is also commutative, monotonic, bounded and idempotent.

4. Families of UPOWAD operators

In the following we analyze different families of UPOWAD operators. The main advantage is that we can consider a wide range of particular cases that can be used in the UPOWAD operator leading to different results. Thus, we are able to provide a more complete representation of the aggregation process.

Remark 1. First of all, we are going to consider the two main cases of the UPOWAD operator that are found by analyzing the coefficient β. Basically, if $\beta = 0$, then we get the uncertain probabilistic distance (UPD), and if $\beta = 1$, the UOWAD operator. Note that when β increases, we are giving more importance to the UOWAD operator and when β decreases, we give more importance to the uncertain probabilistic distance.

Remark 2. Note that the UPOWA operator and all its particular cases are generated through this generalization. This is the case when one of the interval number sets is empty.

Remark 3. Another group of interesting families are the uncertain probabilistic maximum distance, the uncertain probabilistic minimum distance, the step-UPOWAD and the uncertain arithmetic probabilistic distance (UAPD) measures.

- The maximum uncertain probabilistic distance (Max-UPD) is found when $w_1 = 1$ and $w_j = 0$, for all $j \neq 1$.
- The minimum uncertain probabilistic distance (Min-UPD) is found when $w_n = 1$ and $w_j = 0$, for all $j \neq 1$.
- The uncertain normalized distance (UND) is found when $p_i = 1/n$, for all i, and $w_j = 1/n$, for all j.
- The uncertain arithmetic probabilistic distance (UAPD) is found when $w_j = 1/n$, for all j.
- The uncertain arithmetic OWA distance (UAOWAD) is found when $p_i = 1/n$ for all i.
- More generally, the step-UPOWAD is formed when $w_k = 1$ and $w_j = 0$, for all $j \neq k$.

Remark 4. For the median-UPOWAD, if n is odd we assign $w_{(n+1)/2} = 1$ and $w_j = 0$ for all others. If n is even, then we assign $w_{n/2} = w_{(n/2)+1} = 0.5$.

Remark 5. Other families of UPOWAD operators can be constructed by choosing a different weighting vector. For example, when $w_j = 1/m$ for $k \leqslant j \leqslant k+m-1$ and $w_j = 0$ for $j > k+m$ and $j < k$, we obtain the window-UPOWAD operator. Note that k and m must be positive integers such that $k+m-1 \leqslant n$.

Remark 6. Another particular case is the Olympic-UPOWAD. This operator is found when $w_1 = w_n = 0$ and for all others $w_j = 1/(n-2)$. Note that if $n = 3$ or $n = 4$, the olympic-UPOWAD is transformed in the median-UPOWAD and if $m = n-2$ and $k = 2$, the window- UPOWAD is transformed in the Olympic- UPOWAD.

Remark 7. Using a similar methodology, we could develop numerous other families of UPOWAD operators. For more information, refer to [10,15–19,28,34,45].

5. Decision making with the UPOWAD operator

The UPOWAD operator is applicable in a wide range of situations, such as decision making, statistics, engineering and economics. In summary, all of the studies that use the Hamming distance and OWA operator can be revised and extended by using this new approach.

In this section, we show the application of the developed operator through a practical example (adapted from [60]). A car company is desirable to select the most appropriate robot for its manufacturing process. After pre-evaluation, four robots $A_i (i = 1,2,3,4)$ have remained as alternatives for further evaluation. Five criteria are considered as: u_1: Load capacity; u_2: Repeatability; u_3: Speed; u_4: Memory capacity; u_5: Degree of freedom. This company has a group of decision makers from

Table 1
Interval decision matrix-Expert 1.

	u_1	u_2	u_3	u_4	u_5
A_1	[60,70]	[72,80]	[54,67]	[71,82]	[80,90]
A_2	[70,80]	[60,70]	[72,85]	[65,78]	[67,79]
A_3	[70,83]	[57,67]	[72,81]	[69,78]	[73,85]
A_4	[53,60]	[70,80]	[55,70]	[70,85]	[80,93]

Table 2
Interval decision matrix-expert -Expert 2.

	u_1	u_2	u_3	u_4	u_5
A_1	[85,92]	[70,80]	[75,93]	[68,80]	[80,90]
A_2	[83,90]	[78,84]	[83,89]	[79,91]	[78,84]
A_3	[76,89]	[68,84]	[60,70]	[57,70]	[69,87]
A_4	[57,76]	[80,95]	[88,94]	[79,87]	[80,93]

Table 3
Interval decision matrix -Expert 3.

	u_1	u_2	u_3	u_4	u_5
A_1	[76,85]	[80,90]	[68,79]	[75,83]	[79,90]
A_2	[57,70]	[75,83]	[69,74]	[73,86]	[65,73]
A_3	[67,80]	[73,79]	[80,86]	[63,78]	[59,70]
A_4	[82,90]	[68,77]	[74,82]	[80,85]	[80,89]

Table 4
Ideal alternative.

	u_1	u_2	u_3	u_4	u_5
I	[90,100]	[85,95]	[90,100]	[90,100]	[85,95]

Table 5
Collective decision matrix.

	u_1	u_2	u_3	u_4	u_5
A_1	[75.7,84]	[72.6,82]	[67.3,82.4]	[70.3,81.2]	[79.8,90]
A_2	[73.9,83]	[72,79.6]	[76.9,84.8]	[73.6,86.1]	[72.1,80.3]
A_3	[72.4,85.4]	[65.7,77.9]	[67.6,76.5]	[61.8,74]	[68.2,83]
A_4	[60.8,73.8]	[74.6,86.9]	[75.3,84.4]	[76.5,86]	[80,92.2]

three consultancy departments: d_1 is from the production department; d_2 is from the quality inspection department, and d_3 is from the engineering department (whose weighting vector $V = (0.2, 0.5, 0.3)$).

As the environment is very uncertain, the group of experts of the company needs to assess the available information with interval numbers. They give their opinions for each alternative as shown in Tables 1–3. Note that the opinions they provide are interval numbers from 0 to 100. We assume that a value of 100 means that this alternative perfectly fits this criteria and a value of 0 the opposite.

According to the experts' consensus, the company establishes the ideal results that the robot should have in order to be useful for the manufacturing process. The results are presented in Table 4.

First, we aggregate the information about the three experts to obtain a unified decision matrix by using the uncertain averaging (UWA) operator [43]. The results are shown in Table 5.

In this problem, the experts of the company find probabilistic information given as follows: $P = (0.3, 0.3, 0.2, 0.1, 0.1)$ and $\beta = 40\%$. Moreover, the policy of the company is to be very pessimistic whenever the future results are not clear. Therefore, they decide to manipulate the probabilities by using the following OWA weighting vector $W = (0.1, 0.2, 0.2, 0.2, 0.3)$.

With this information, it is now possible to aggregate the available information in order to take a decision. The method consists in comparing the available robots with the ideal one by using the UPOWAD operator and its particular cases. We are able to provide a more complete picture to the decision maker because we are able to consider different future scenarios. Due to our uncertainty, we do not know which scenario is the correct scenario. Therefore, the representation of different

Table 6
Aggregated Results.

	Max-UPAD	Min-UPD	UND	UPD	UOWAD	UAPD	UAOWAD	UPOWAD
A_1	17.44	12.23	14.47	16.63	12.97	14.68	13.87	14.08
A_2	15.40	10.60	14.77	15.15	14.50	14.88	14.66	14.77
A_3	22.81	17.73	19.75	21.71	18.48	19.32	19.24	18.81
A_4	20.52	11.48	13.95	16.92	11.57	15.12	12.99	14.16

Table 7
Ordering of the robots.

	Ordering		Ordering
Max-UPD	$A_2 \succ A_1 \succ A_4 \succ A_3$	UOWAD	$A_4 \succ A_1 \succ A_2 \succ A_3$
Min-UPD	$A_2 \succ A_4 \succ A_1 \succ A_3$	UAPD	$A_1 \succ A_2 \succ A_4 \succ A_3$
UND	$A_4 \succ A_1 = A_2 \succ A_3$	UAOWAD	$A_4 \succ A_1 \succ A_2 \succ A_3$
UPD	$A_2 \succ A_1 \succ A_4 \succ A_3$	UPOWAD	$A_1 \succ A_4 \succ A_2 \succ A_3$

particular cases that could happen (from the minimum to the maximum) seems to be useful for gaining a complete picture of the different future situations. Thus, the decision maker knows the results that can be obtained with each alternative and thus, select the one that seems to be in closest accordance with his interests. Note that this analysis is done in uncertainty so we do not know the correct answer until the future becomes the present.

In this example, we consider the Max-UPD, the Min-UPD, the UND, the UPD, the UOWAD, the UAPD, the UAOWAD and the UPOWAD operators. The optimal choice would be the alternative closest to the ideal. The results are shown in Tables 6.

As we can see, depending on the uncertain distance aggregation operators used, the optimal choice is different. Note that in this problem the maximum uncertain probabilistic distance is the most pessimistic aggregation because it considers only the highest distance, that is, the worst characteristic of an alternative. On the other hand, the minimum uncertain probabilistic distance is the most optimistic one. The UND is a neutral aggregation because it gives the same weights to all the characteristics. The uncertain probabilistic aggregation distance is also neutral but weights the characteristics according to some probabilistic information. The UOWAD operator assumes that we are in an uncertain environment where we can only aggregate the information considering the attitudinal character of the decision maker. The UAPD combines probabilistic information with the UND. The UAOWAD operator uses the UOWAD operator and the UND in the same aggregation. The UPOWAD operator combines probabilistic information with the UOWAD operator.

If we establish a ranking of the alternatives, a common situation when considering more than one selection, we get the following results shown in Table 7. Note that the best choice is the one with the lowest distance.

As we can see, depending on the aggregator operator used, the ranking of the robots may be different. Note that the main advantage of using the UPOWAD operator is that it can consider a wide range of particular distance measures such as the Max-UPD, the Min-UPD and the UOWAD operator. Due to the fact that each particular family of UPOWAD operator may give different results, the decision maker will select for his decision the one that is closest to his interests. However, by looking to this analysis he will be able to see the results and optimal decisions in other potential situations that may occur in the future.

6. Conclusions

We have introduced the UPOWAD operator as an aggregation operator that uses the main characteristics of the probability, the OWA operator, the Hamming distance and uncertain information represented in the form of interval numbers. The main advantage of this operator is that it provides more complete information because it represents the information in a more complete way considering the maximum and the minimum results that can occur. Moreover, it includes many different types of uncertain distance measures and aggregation operators, such as the UPOWA, the UAPD and the UOWAD operator.

We have also presented an application of the new approach to a group decision making problem concerning the selection of robots in a manufacturing process. We have seen that the UPOWAD is very useful because it represents very well the uncertain information by using interval numbers. The main advantage of the UPOWAD operator in decision making is that it shows a lot of different scenarios that could happen depending on the particular type of UPOWAD operator used in the problem. The main problem that we identify is that we do not have one model that yields the best decision, because we are dealing with uncertainty. Obviously, given these types of problems, the best way to assess information is through a general model that includes different methods in the same formulation, although it cannot identify one method with the best decision. Therefore, this general model (UPOWAD) at least provides potential results that may occur in the decision problem, so that the decision-maker knows these different results could happen and thus selects the one in accordance with his/her interests.

In future research we expect to develop further extensions by adding new characteristics in the problem such as the use of order-inducing variables. We will also consider other decision making applications such as human resource management and financial management.

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