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A Linguistic Hierarchical Evaluation Model for Engineering Systems

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Abstract

Before implementing a design of a large engineering system different design proposals are evaluated and ranked based on subjective assessments of different criteria. The knowledge about these criteria might be vague and incomplete. So, to deal with this kind of information we shall use linguistic approaches and Dempster-Shafer theory of evidence. In this chapter we propose an evaluation model based on the criteria of Safety and Cost. The safety assessments will be obtained using the fuzzy rule-based evidential reasoning (FURBER) approach and the cost assessments are supplied by the experts. Both subjective criteria are usually assessed in different utility spaces. The aim of this contribution is to evaluate the different design options by means of a decision model applying a linguistic hierarchical process to avoid loss of information.

Key words: Engineering systems, cost, safety, evaluation, linguistic information

1. Introduction

In the design of large engineering products such as offshore topsides, and offshore cranes, an efficient design may be evaluated and selected by means of Multi-Criteria Decision Making (MCDM) techniques. The decision of implementing a design will depend on that satisfies technical and economical constraints. In this chapter is proposed a linguistic evaluation model that takes into account the criteria of *Safety* and *Cost*. Hence, subjective safety and cost assessments can be studied together to determine the best risk reduction action and to choose the best design/operation option. So, multiple safety analysts can provide their subjective judgments for each design option on both cost and safety aspects.

Different safety assessment approaches may be difficult to use in situations where there is a lack of information, past experience, or ill-defined situation in risk analysis [11]. Therefore, linguistic descriptors, such as, "*Likely*", "*Impossible*", are used to describe an event due to the fact they are used commonly by engineers and safety analysts. Hence, the use of the fuzzy linguistic approach [14] is a good model to analyze the safety of engineering systems with incomplete information. Also the estimation of the cost is a ill-defined situation, therefore the use the linguistic approach is adequate too.

In engineering safety analysis, intrinsically vague information may coexist with conditions of "lack of specificity" originating from evidence not strong enough to completely support a hypothesis but only with degrees of belief or credibility. Dempster-Shafer (D-S) theory of evidence [7,9] based on the concept of belief function. D-S theory enlarges the scope of traditional probability theory, describes and handles uncertainties using the concept of the degrees of belief, which can model incompleteness and ignorance explicitly. Besides, the D-S theory also shows great potentials in multiple attribute decision analysis under uncertainty [13].

The aim of this paper is to develop a linguistic decision model that evaluates different design options for a large engineering system according to safety and cost criteria. To do so, we propose:

(i) An evaluation framework to assess the safety and cost criteria.
  - Safety will be assessed based on fuzzy logic and the evidential reasoning approach, referred to as a FURBER approach [5], which is based on the RIMER approach proposed recently in [12].
  - The synthesis of the safety assessments for each option is expressed and implemented using a linguistic 2-tuple scheme [3].
  - The cost assessments of each design option will be synthesized based on the assessments of each cost factor that are supplied directly by the experts in terms of linguistic labels. The assessments of each criterion are conducted in different utility spaces from each other.

(ii) An evaluation model based on a Multi-Expert MCDM process.
  - These assessments are the input values for a Multi-Expert Multi-Criteria Decision Making (MEMC-DM) problem defined in a multi-granular linguistic domain.
  - In the evaluation process the cost and safety assessments will be unified in a common utility space by means of linguistic hierarchies [4] and the linguistic 2-tuple representation model and after combined to obtain a degree of suitability for each design option to choose the best one.

In order to do so, this chapter is structured as follows: in Section 2 we make a brief review of linguistic tools. In Section 3 we describe the evaluation framework for safety and cost modelling of large engineering systems. In Section 4 it will be presented the application of the Hierarchical linguistic decision model to evaluate the design options. And finally, some conclusions are pointed out.

2. Linguistic Background

In this section we shall review some core concepts about linguistic information. We review briefly the *2-tuple Linguistic model* and the *Linguistic Hierarchies.*

2.1. *The 2-tuple Linguistic Model*

This model was presented in [3], for overcoming the drawback of the loss of information presented by the classical linguistic computational models: (i) The semantic model [1], (ii) and the symbolic one [2]. The 2-tuple fuzzy linguistic representation model is based on the symbolic method and takes as the base of its representation the concept of Symbolic Translation.

Definition 1. The Symbolic Translation of a linguistic term $s_i \in S = \{s_0, \cdots, s_g\}$ is a numerical value assessed in $[-0.5, 0.5)$ that supports the "difference of information" between an amount of information $[0, g]$ and the closest value in $\{0, \cdots, g\}$ that indicates the index of the closest linguistic term in $S(s_i)$, being $[0, g]$ the interval of granularity of S.

From this concept a linguistic representation model is developed, which represents the linguistic information by means of 2-tuples (s_i, α_i), $s_i \in S$ and $\alpha_i \in [-0.5, 0.5)$. This model defines a set of functions between linguistic 2-tuples and numerical values.

Definition 2. Let $S = \{s_0, \cdots, s_g\}$ be a linguistic term set and $\beta \in [0,]$ a value supporting the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:

$$\Delta : [0, g] \to S \times (-0.5, 0.5)$$

$$\Delta(\beta) = (s_i, \alpha), with \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-0.5, 0.5) \end{cases} \tag{1}$$

where s_i has the closest index label to "β" and "α" is the value of the symbolic translation.

Proposition 1. Let $S = \{s_0, \cdots, s_g\}$ be a linguistic term set and $(s_i, \alpha_i$ be a linguistic 2-tuple. There is always a Δ^{-1} function, such that, from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g]$.

Proof. It is trivial, we consider the following function:

$$\Delta^{-1} : S \times [-0.5, 0.5) \to [0, g]$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta \tag{2}$$

Remark 1. *From Definitions 1 and 2 and Proposition 1, the conversion of a linguistic term into a linguistic 2-tuple consist of:* $s_i \in S \to (s_i, 0)$

In addition this model has a computational technique based on the 2-tuples were presented in [3].

2.2. *Linguistic Hierarchies*

The hierarchical linguistic contexts were introduced in [4] to improve the precision of the processes of Computing with Words in multi-granular linguistic contexts, that it is the aim of this contribution.

A Linguistic Hierarchy is a set of levels, where each level represents a linguistic term set with different granularity to the remaining levels. Each level is denoted as $l(t, n(t))$ being,

- t a number that indicates the level of the hierarchy.
- $n(t)$ the granularity of the term set of the level t.

The levels belonging to a linguistic hierarchy are ordered according to their granularity, i.e., for two consecutive levels t and $t+1$, $n(t+1) > n(t)$. Therefore, the level $t+1$ is a refinement of the previous level t.

From the above concepts, we define a linguistic hierarchy, LH, as the union of all levels t:

$$LH = \cup_t l\left(t, n\left(t\right)\right)$$

Given a LH, we denote as $S^{n(t)}$ the linguistic term set of LH corresponding to the level t of LH characterized by a granularity of uncertainty $n(t)$:

$$S^{n(t)} = \{s_0^{n(t)}, \ldots, s_{n(t)-1}^{n(t)}\}$$

Generically, we can say that the linguistic term set of level $t+1$ is obtained from its predecessor as:

$$l(t, n(t)) \longrightarrow l(t+1, 2 \bullet n(t) - 1)$$

A graphical example of a linguistic hierarchy can be seen in Figure 1:

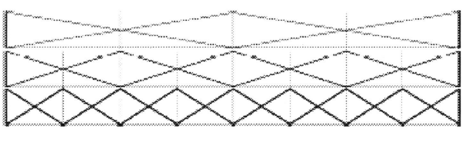

Fig. 1. Linguistic Hierarchy

In [4] were developed different transformation functions between labels of different levels without loss of information.

Definition 3. Let $LH = \cup_t l(t, n(t))$ be a linguistic hierarchy whose linguistic term sets are denoted as $S^{n(t)} = \{s_0^{n(t)}, \ldots, s_{n(t)-1}^{n(t)}\}$, and let us consider the 2-tuple linguistic representation. The transformation function from a linguistic label in level t to a label in level t' is defined as:

$$TF_{t'}^{t} : l(t, n(t)) \longrightarrow l(t', n(t'))$$

$$TF_{t'}^{t}(s_i^{n(t)}, \alpha^{n(t)}) = \Delta_{n(t')}\left(\frac{\Delta_{n(t)}^{-1}\left(s_i^{n(t)}\right) \bullet (n(t')-1)}{n(t)-1}\right) \quad (3)$$

Proposition 2. The transformation function between linguistic terms in different levels of the linguistic hierarchy is bijective:

$$TF_{t}^{t'}(TF_{t'}^{t}(s_i^{n(t)}, \alpha^{n(t)})) = (s_i^{n(t)}, \alpha^{n(t)})$$

3. Evaluation Framework for Engineering Systems

In this section we show briefly how are the assessments for safety using the FURBER approach [5,12] and how are the cost assessments provided by the experts.

3.1. *Safety Evaluation Framework*

A generic framework for modelling system safety estimate using FURBER approach and for safety synthesis is outlined, more details see [5,12].

Step #1: Identification of causes/factors: it can be done by a panel of experts during a brainstorming session at the early concept design stages.

Step #2: Identify and definite fuzzy input and fuzzy output variables (i.e., safety estimates)

The three fundamental parameters used to assess the safety level of an engineering system on a subjective basis are the failure rate (**FR**), consequence severity (**CS**) and failure consequence probability (**FCP**). Subjective assessments are more appropriate for analysis using these three parameters as they are always associated with great uncertainty.

Safety estimates is the only output fuzzy variable used to produce safety evaluation for a particular cause to technical failure. This variable is described linguistically, which is described and determined by the above parameters. In safety is common to express a safety assessment in the following linguistic term set [10], that we note as, S_S:

$$S_S = \{Poor, Low, Average, High, Good\}$$

which are referred to as safety expressions.

Step #3: Construct a fuzzy rule-base

Fuzzy logic systems are knowledge-based or rule-based ones constructed from human knowledge in the form of fuzzy IF-THEN rules. We shall use to build our fuzzy rule base a linguistic term set with seven labels for **failure rate** (i.e., $J_1 = 7$); five labels for **consequence severity** (i.e.,$J_2 = 7$), seven labels for **failure consequence probability** (i.e.,$J_3 = 7$). Therefore, being L the total number of rules, in this case we use a sample of $L = 245$ rules [5].

Step #4: Fuzzy rule-base inference mechanism.

Suppose a fuzzy rule-base with the belief structure is given by $R = \{R_1, \cdots, R_L\}$. The k^{th} rule can be represented as follows:

R_k : IF U is A^k THEN **safety estimate** is D with belief degree Y_k

where U represents the antecedent attribute vector (**FR**, **CS**, **FCP**), A^k the packet antecedents $\{A_1^k, A_2^k, A_3^k\}$, D the consequent vector $(D_1, \cdots, D_N)$, Y_K the vector of the belief degrees $(Y_{ik}, \cdots, Y_{nk})$ and $k \in 1, \cdots, L$.

Once a rule-base is built up its knowledge can be used to perform the inference procedure. In order to reach a safety assessment the fuzzy reasoning system expresses the safety estimates $S(e_i(a_l))$ as follows for the assessment done by the i^{th} expert on the l^{th} potential cause to a technical failure:

$$S(e_i(a_l)) = \Big\{ (Poor; \Theta_{1i}^l); (Low; \Theta_{2i}^l); (Average; \Theta_{3i}^l); (High; \Theta_{4i}^l); (Good; \Theta_{5i}^l) \Big\}$$

where e_i represents the i^{th} expert $(i = 1, \cdots, p)$ and a_l represents the l^{th} $(l = 1, \cdots, q)$ potential cause to a technical failure. Θ_{ti}^l represents the belief degree to which the safety of a_l is believed to be assessed to D_t by the expert e_i. The inference procedure is based on fuzzy rule-base and evidential reasoning approach, referred to as a fuzzy rule-based evidential reasoning approach - FURBER approach [5]. The final result is still a belief distribution on safety expression, which gives a view about the safety level for a given input.

In this phase for the synthesis purpose, we transform the safety estimate into a linguistic 2-tuple, i.e., transform the distribution assessment $S(e_i(a_l))$ on the S_S into linguistic 2-tuples over the S_S. A function X_i^l is introduced that transforms a distribution assessment in a linguistic term set S_S into a linguistic 2-tuple in S_S:

$$\chi_i^l : S\left(e_i\left(a_l\right)\right) \rightarrow S_S \times [-0.5, 0.5)$$

$$\chi_i^l \left(\left\{\left(s_t; \Theta_{ti}^l\right), t = 0, \ldots, g-1\right\}\right) = \Delta\left(\frac{\sum_{t=0}^{g} t\Theta_{ti}^l}{\sum_{t=0}^{g} \Theta_{ti}^l} = \beta_i^l\right) \quad (4)$$

3.2. *Cost Modelling*

Cost and safety are two of the most important features for the engineering systems, but usually they are conflicting because higher safety leading to higher costs. The cost incurred for safety improvement associated with a design/operation option is usually affected by many factors that often have large uncertainties of estimation. Therefore, it may be more appropriate to model the cost incurred in safety improvement associated with the design option on a subjective basis.

In the literature [10,11] these assessments are described by means of linguistic values. In our proposal we are interested in develop a model without loss of information using the linguistic hierarchies. To do so and due to the safety is assessed in a linguistic term set with five labels, the experts will express the cost assessments in a linguistic term set with nine labels, S_C. We propose the following term set (triangular shaped and symmetrically distributed):

$S_C = \{None, VeryLow, Low, ModeratelyLow, Average, ModeratelyHigh, High, VeryHigh, Unacceptable\}$

In our proposal the experts provide directly the cost assessments by means of labels in S_C.

Remark 2. *Cost assessments have a different interpretation that safety assessments, i.e., high cost indicates low suitability of the design option.*

4. Evaluation Model: Ranking Engineering Design Options

The aim we pursue solving this problem is to choose the most suitable design option for an engineering system taking into account features from safety and cost. So far, the assessments of safety are assessed in S_S while the assessments of the cost are assessed in S_C. Then to evaluate and rank the options we shall apply the multi-granular linguistic decision model presented in [4] in order to solve our problem. This model uses linguistic hierarchies to manage decision making problems defined in multi-granular linguistic domains without loss of information, that it is a very important feature in the development of engineering systems. After choosing a LH for the evaluation framework the evaluation model will consist of two phases:

- Aggregation phase: it combines safety and cost assessments to obtain an overall suitability assessment for each option. This phase has two steps:
  (i) Normalization process: it unifies the multi-granular linguistic information.
  (ii) Aggregation process: it obtains an overall value of suitability for each design option.
- Exploitation phase: it ranks the different design options according to assessments obtained in the aggregation phase by means of a choice degree.

4.1. *Safety and Cost Problem Modelled by means of Linguistic Hierarchies*

So far, we have the Safety and Cost assessments of each design option expressed by means of linguistic values assessed in different linguistic utility spaces with five and nine labels respectively. Therefore, we have to choose a linguistic hierarchy, LH, for modelling our problem. We shall choose a LH that contains levels with five and nine labels respectively (see Fig. 2):

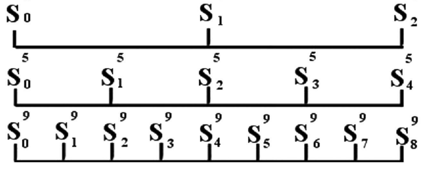

Fig. 2. Linguistic Hierarchy 3,5 and 9 labels

4.2. *Evaluation based on a Decision Process*

Here, we describe in detail the steps of the evaluation model used to solve our problem that is modelled as a Multi-Expert Multi-Criteria Decision Making problem where each expert i, provides assessments for the cost and from his/her opinions are synthesized assessments for the safety (See Table 1):

Table 1
Expert's assessments

Design Options		
Expert i	Safety	Cost
O_1	(s_{i1},α)	(c_{i1},α)
...	...	...
O_n	(s_{in},α)	(c_{in},α)

Where (s_{ij},α) are the safety assessments synthesized from the opinions of the expert e_i for the design option o_j, i.e., estimated based on the fuzzy rule-based system, and then synthesized to obtain the safety assessment of the system by means of linguistic 2-tuples in the linguistic term set, S_S. While (c_{ij},α) are the overall cost assessment obtained aggregating the cost of the different cost factors, provided by the expert e_i for the design option O_j, assessed in the linguistic term set S_C (These values are transformed into 2-tuples according to Remark 1).

4.2.1. *Aggregation Phase*

In this phase the information is combined to obtain collective preference values for each design option according to the assessments of the different experts and criteria. This model combines the multi-granular linguistic information in two steps.

(i) Normalization Process:

We are dealing with multi-granular linguistic information, to manage it the model unifies it in a common utility space called, Basic Linguistic Term Set (BLTS). We propose as the common utility space for expressing the overall utility of each design option the following linguistic term set, with five labels:

$$S_T = \{SlightlyPreferred, ModeratelyPreferred, Average, Preferred, GreatlyPreferred\}$$

Remark 3: *during the aggregation process is used the notation, s_i^5, to refer to the aggregated values to avoid misunderstandings with cost meaning.*

In this case our model chooses the BLTS as the second level of the linguistic hierarchy shown in the (Fig. 2) which granularity is five labels.

Once we have chosen the common utility space to express the preferred design options we have to transform the safety and cost assessments to the BLTS. Let's suppose the following assessments (See Table 2):

The multi-granular information is unified by means of the transformation function between the levels of the hierarchy (Def. 3). The safety assessments of safety are already expressed in the BLTS, while the cost assessments are

Table 2
Linguistic Safety and cost assessments in S_S

	Safety			Cost		
Opts	Experts					
	E#1	...	E#M	E#1	...	E#M
O_1	(poor,0.2756)	...	(poor,0)	(ModerateHigh,0)	...	(Average,0)
$\vdots$		...			...	
O_n	(Low,-0.459)	...	(poor,0.445)	(High,0)	...	High,0)

unified by means of $TF_5^9(c,\alpha)$. This process is applied to all the experts opinions. So, the safety and cost assessments of the expert i are expressed by means of linguistic 2-tuples in the common utility space, BLTS (See table 3):

Table 3
Linguistic Safety and Cost assessments expressed in the BLTS

	Safety			Cost		
Opts	Experts					
	E#1	...	E#M	E#1	...	E#M
O_1	$\left(s_0^5, 0.2745\right)$	...	$\left(s_0^5, 0\right)$	$\left(s_3^5, -0.5\right)$	...	$\left(s_2^5, 0\right)$
$\vdots$		...			...	
O_n	$\left(s_1^5, -0.4594\right)$	...	$\left(s_0^5, 0.4453\right)$	$\left(s_3^5, 0\right)$	...	$\left(s_3^5, 0\right)$

(ii) Aggregation Process: it combines the cost and safety assessments to obtain a global value for each design option in a two-step process:
   (a) Obtain a global value for cost and safety of each design option expressed in a linguistic value in the BLTS. To do so, we can use different aggregation operators for linguistic 2-tuples defined in [3]. In this paper we use the weighted average operator to obtain a global value for cost and safety, (See Table 4):

Table 4
Global assessments for Safety and Cost

	Safety	Cost
O_1	$\left(s_1^5, -0.3\right)$	$\left(s_3^5, -0.12\right)$
...	...	...
O_n	$\left(s_0^5, 0.34\right)$	$\left(s_3^5, 0.23\right)$

   (b) Obtain an evaluation value for each design option in the BLTS. To do so, we have a set of pairs of assessments $\{(s_i, \alpha), (c_i, \alpha)\}$ for each design option, the applying a weighted aggregation operator taking into account the remark 2 the aggregated value for each design option is obtained with the following expression:

$$W_{AM}\left((s_i,\alpha),(c_i,\alpha)\right)=\Delta\left(\Delta^{-1}(s,\alpha)\cdot w+\Delta^{-1}\left(Neg\left(c,\alpha\right)\right)\cdot(1-w)\right)$$

Where $Neg(c_i,\alpha)$ is the assessment for the cost of the design option i taking into account its decreasing interpretation and (s_i,α) is the assessment for the safety of the option i. And w is the weight for the safety assessment and $1-w$ the weight for the cost assessment. Let suppose a value of $w=0.5$ then from Table 4:

Table 5
Design Options Utility Assessments

Design Options	
O_1	(Moderated Preferred,0.44)
$\vdots$	$\vdots$
O_n	(Moderated Preferred, 0.05)

Now we have got an overall value of each design option expressed by means of a linguistic 2-tuple in S_T for each expert. To obtain a global assessment for each option we shall apply another aggregation operator to the global assessments of all experts. We could consider that all the experts are equally important or we could assign different weights to each experts.

4.2.2. *Exploitation Phase*

Finally the decision process applies a choice degree to obtain a selection set of alternatives. Different choice functions has been proposed in the choice theory literature [6,8]. The choice functions rank the alternatives according to different possibilities and from the ranking obtained the best one/s.

In our problem the information is expressed by linguistic 2-tuples that have defined a total order over itself. In our problem we shall rank the results using this order. Let's suppose an only expert then from Table 7 we shall choose as the best design option: O_1

5. Conclusions

The evaluation of different designs before implementing a large engineering system is a common task. In this paper we have proposed an evaluation model based on an MEMCDM problem that evaluates the engineering systems according to its Safety and Cost. The main advantage of this evaluation model is that it manages multi-granular linguistic information without loss of information.

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